SOROBAN ABACUS HANDBOOK

**The Soroban / Abacus Handbook
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Rev 1.03 - June 8, 2004**

INTRODUCTION

The Abacus (or Soroban as it is called in Japan) is an ancient mathematical instrument used for calculation. The Abacus is one of the worlds first real calculating tools – and early forms of an Abacus are nearly 2500 years old. The word Abacus is derived from the Greek "Abax" meaning counting board and the original types of Abacus were stone slates with dust covering them and a stylus used for marking numbers. Later this evolved into a slate with groves where rocks or other counters would be placed to mark numbers. Later it finally evolved into a framed device with beads sliding along bamboo rods.

I have always been fascinated by the abacus – and have recently taken up the study of this tool. I have practiced on both a Chinese Abacus (called a *Suan Pan*) and on a Japanese *Soroban*. The modern Chinese abacus has been in use since about the 14th century. The Japanese Soroban has been in use since at least the 16th century. I much prefer the Japanese Soroban as a more aesthetically pleasing and a more efficient calculating instrument. There are some key differences between the two types of instruments.

Here is a picture of a traditional Chinese Abacus. As you can see this instrument has 2 beads above the reckoning bar and 5 beads below it.
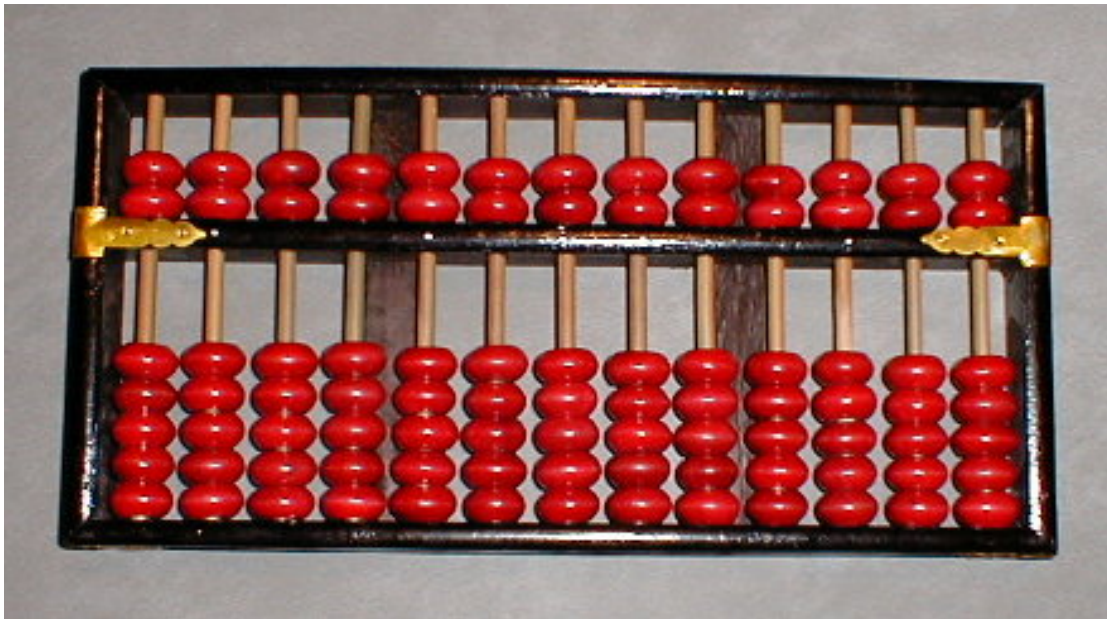
And here is a picture of a traditional Japanese Soroban. Here we have a streamlined instrument with 1 bead above the reckoning bar and 4 below it.
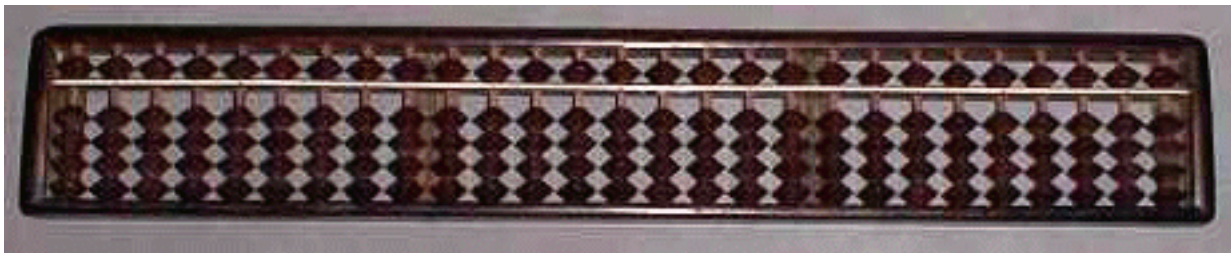
Originally the Japanese Soroban looked much like the Chinese Abacus (5 beads below, 2 beads above) but it was simplified around 1850 and reduced to a single bead above the reckoning bar and later in 1930 to just 4 beads below it.

It doesn't matter which type you use – both have the same procedure for recording numbers and performing addition, subtraction, multiplication and division. The Chinese abacus is capable of counting 16 different numbers from 0 to 15 on each individual rod which was useful since their units of weight were (are?) measured in 16ths. For westerners – this is not very useful unless you want to do calculations in 16ths of an inch or maybe hexadecimal (for computer programs which is base 16). Of course you don't have to use all the beads – so you can always just represent numbers 0-9 which is most useful for our purposes. The Japanese Soroban has been streamlined for the Hindu-Arabic number system and each rod can represent one of 10 different numbers (0-9) and has no wasted beads for our decimal calculations. Read on to discover how numbers are represented on the abacus frame.

I have researched both types and studied different methods for performing operations on the

board. I will try to instruct and teach only the best methods that I have found. Generally, these are the methods described by Takashi Kojima in his excellent book "The Japanese Abacus – It's Use and Theory" published first in the 1950's and later reprinted. I assume the reader wants to know the best prescribed procedures for learning how to use the abacus. I will try to take no shortcuts – and hope that this handbook serves as a fine introduction to the marvelous world of the Abacus. One might ask why anyone should bother learning how to use the Abacus with the advent of such cheap calculators. The answers will be different for everyone – but for me it was a desire to learn and understand this ancient skill and to become skilled myself with this fascinating tool. I also find the practice on the Soroban very relaxing – often helping me unwind after a day at work. I suppose it is as much a hobby as anything else. With practice and the help of this handbook, you can master the Abacus! If you don't have an instrument, you can find proven suggestions at the end of this handbook for tracking one down or making one yourself!

USING THE ABACUS

The **beads** on the abacus slide up and down on what we call **rods** which are divided horizontally by a **reckoning bar** (some call this the **beam**). In general, the single row of beads above the reckoning bar (called "Heaven Beads") are worth 5. The beads below the reckoning bar (called "Earth Beads") are worth 1. A bead is said to "have value" when it is pushed towards the middle reckoning bar and "loses value" when moved away from the middle reckoning bar. Forming a number on the abacus is very simple – simply move a bead towards the reckoning bar for it to "have value". Let's look at this virtual abacus frame and "read" the number.
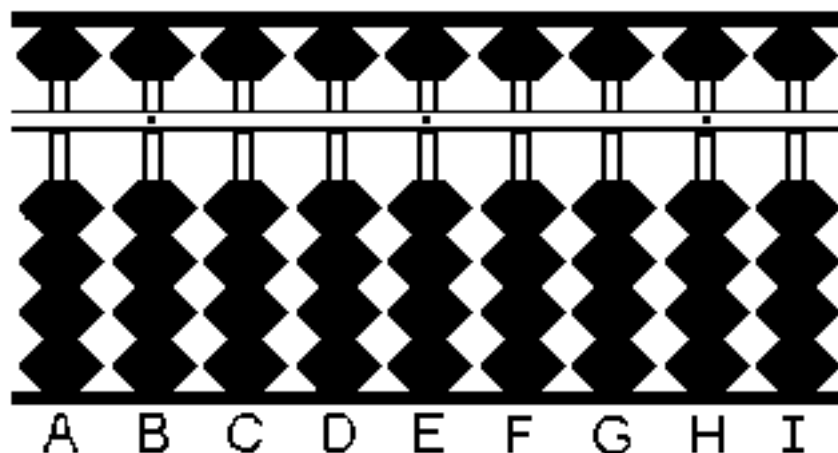
Since no beads are touching the middle reckoning bar, we read a number of 0 all the way across the abacus. This is a "cleared" frame and it is how you normally start every math problem (similar to a reset on a modern calculator). To clear a Soroban – you simply tilt it

towards you so that all the beads are pulled by gravity towards the bottom of the frame (this clears the lower beads only!) and then level the instrument again and make a sweeping motion of your index finger between the reckoning bar and the upper row of beads – this will cause them to "push" upwards away from the reckoning bar. It's almost as if you were splitting the beads from the reckoning bar with a parting sweep of your finger. You could just move them up individually with a flick of your index finger – but the sweeping method from left to right is very fast and efficient once you get the hang of it.

Notice the little dots on the reckoning bar. We use these dots on every third column to designate a "unit" rod. You should place your number so that the unit portion falls on this rod (for example ,in the number 1234 the value 4 is the "units", 3 is the "tens", 2 is the "hundreds", etc). Use rods to the right of this unit rod as tenths, hundredths, and rods to the left as tens, hundreds, thousands (which also has a unit dot – very convenient!). It doesn't matter which dot you choose to be the unit rod – so long as it is marked with a dot for easy reference. Numbers can be formed on the abacus anywhere you want – and with some problems (like multiplication and division) there will be two or three sets of numbers on the abacus frame. Do not think of the dot as a decimal point – that might get confusing as to whether that rod is representing the value just before the decimal point or just after it. It is a unit rod – and rods to the right have a factor of 10 less value and rods to the left have a factor of 10 more. For example, lets assume in the diagram above we chose E as our unit rod. We would then have:

Rod	What it Represents
Rod A	Ten Thousands (10000's)
Rod B	Thousands (1000's)
Rod C	Hundreds (100's)
Rod D	Tens (10's)
Rod E	Units (1's)
Rod F	Tenths (0.1's)
Rod G	Hundredths (0.01's)
Rod H	Thousandths (0.001's)

| **Rod I**	Ten Thousandths (0.0001's)

Before we move on, you have just enough information to understand the way numbers are formed on the frame. I will take the time here to give you some tips for Soroban mastery – some of these won't make sense until later in the handbook – but you can start to apply them as you learn the full techniques for the different operations we will perform on the Abacus.

- Always work numbers on the Abacus from left to right. It's the most efficient way. Don't fall into old habits of trying to add and subtract starting at the right (as you would with pencil and paper).
- When using the Abacus, place it on a level surface – steadying it with your left hand if necessary and work with your right hand (otherwise you partially block and cover numbers needlessly as you work with it).
- Use two fingers for bead manipulation only – the thumb is always used when moving one or more earth beads up to the reckoning bar. The index finger is used for everything else (moving earth beads back away from the reckoning bar and moving heaven beads to and from the reckoning bar). Use just enough force to move the beads – do not slam them which can disrupt beads on neighboring rods.
- When performing addition, always finish moving beads on the current rod before dealing with any carry to the tens rod (which is always the rod to the left of the one you are working on – it will always have 10x the value of the current rod).
- When performing subtraction, always borrow from the tens rod before finishing moving beads on the current rod (which is always the rod to the left of the one you are working on – it will always have 10x the value of the current rod).

ADDITION

Let's put a real number on the Abacus. Let's put the number **21** on the frame. We conveniently choose rod H as our unit rod and form the number.
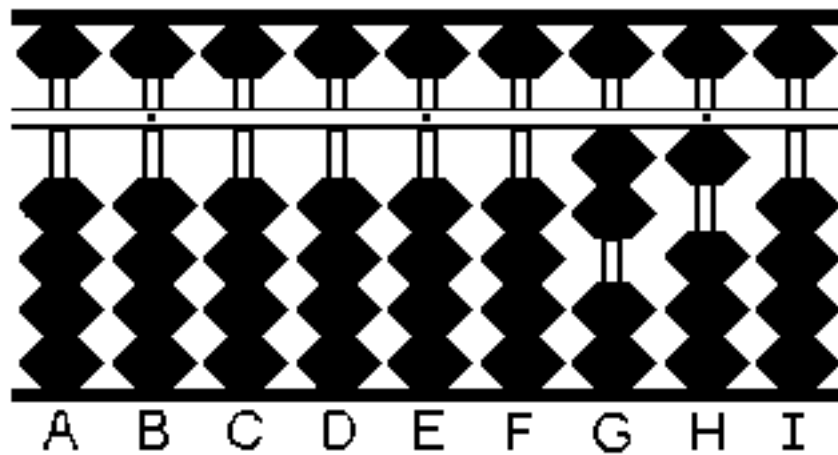
Here we have placed the value of 2 on rod G (by moving 2 earth beads worth 1 each towards the reckoning bar) and the value of 1 on rod H (by moving a single earth bead worth 1 each towards the reckoning bar). I will henceforth use shorthand for this and simply say "**Place 21**" which means to place 21 on the frame (remember to observe the unit rod when you place the numbers – it will help you to remember the size and position of the number more easily as the calculations get more complicated).

It's important to know that when you enter this number on the frame, you should enter it from left to right. Numbers are read and spoken from left to right and so it's much more efficient to enter them on the frame in the same manner. Again, be sure your number ends up on a unit rod for convenience.

Now let's add **6** to this number.
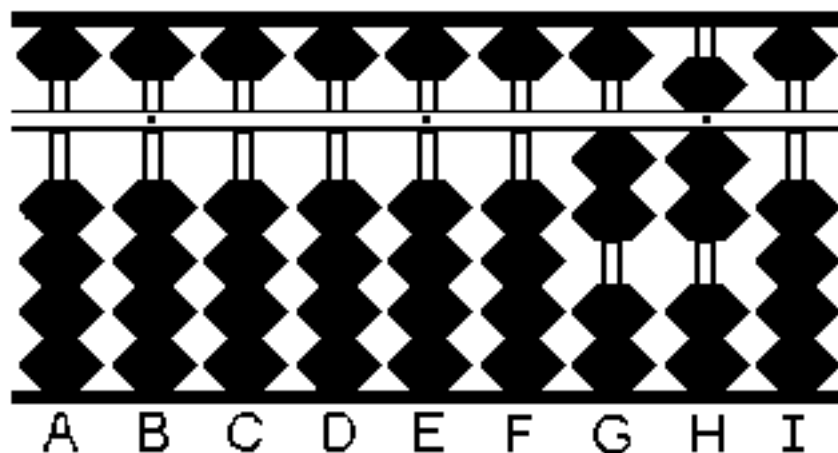
As you can see, we added 6 to the units place of the original number (21) on the frame. We do this by moving 6 worth of beads towards the reckoning bar for rod H. We don't have enough earth beads (worth 1) to do it so we must take a single heaven bead (worth 5) and a single earth bead (worth 1) to make the 6 we need. Using the proper fingering techniques described before – you use your thumb to move up the singe earth bead and your index finger to move down the single heaven bead. This can be done in one motion – almost a "squeezing" effect

that is very efficient. After moving those beads towards the reckoning bar (henceforth in this document called "**Add 6**") you can now "read" the value resulting on the frame. Rod G has a value of 2 and rod H (our unit rod) has a value of 7. The answer, therefore, is 27. There, you've just added your first numbers on the Soroban!

Now, let's get tricky! We will take this resulting answer (27) and add **15** to it. Starting with the 1 in "15" we want to put this on Rod G – so we simply "Add 1" by moving a single bead towards the reckoning bar. See the following resulting frame which now contains the value of 37 (not the final answer, just the intermediate value after adding the 1 in "15").
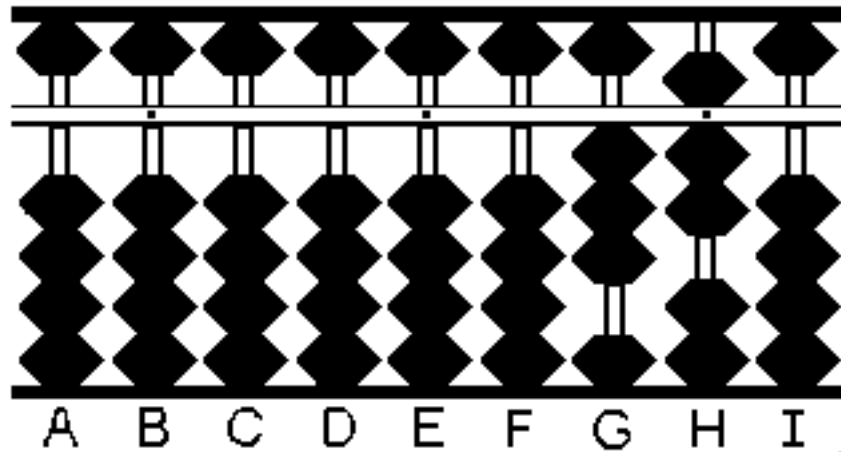

And now we must add the 5 of the "15" (remember, we are working from left to right with these numbers) and so we must "Add 5" on the unit rod (H). Wait! We don't have 5 beads to work with – so instead we "Subtract 5" and "Add 10" which involves a shift to the next column up (to the left which is 10 times the value of the current rod). This is a key concept on the Abacus – we didn't have enough to "Add 5" to our unit column so we must instead "Subtract 5" on the unit rod and then "Add 10" (by moving a single earth bead worth 1 in the next column up – in this case rod G). In this case, always work with the current rod first – moving a single heaven bead away from the reckoning bar on H and then moving up a single earth bead on G.

This results in the following frame:
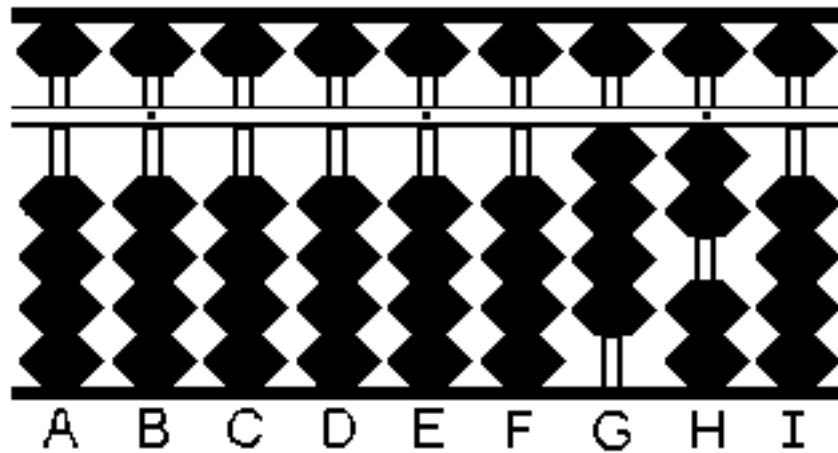
As you can see, the resulting number on GH is **42** which is the result of adding 15 to 27. It is imperative to remember that when wanted to add the 5 to the unit rod H, you did not mentally say "Well… there is 7 already on the rod, and I'm adding 5 so that is 12 – I must somehow form 12 on the frame!". That line of thinking will make abacus processing very slow – the right way to do it is to know that you must "Add 5" on H – if you lack the beads, you must "Add 1" on the next highest rod and "Subtract 5" on the unit rod which accomplishes the same thing. This sort of mechanical processing is quick to learn and allows for incredible efficiency when working with the Soroban.

The same borrowing processing results even within a single rod. Let's look at the following number **14** on the frame:
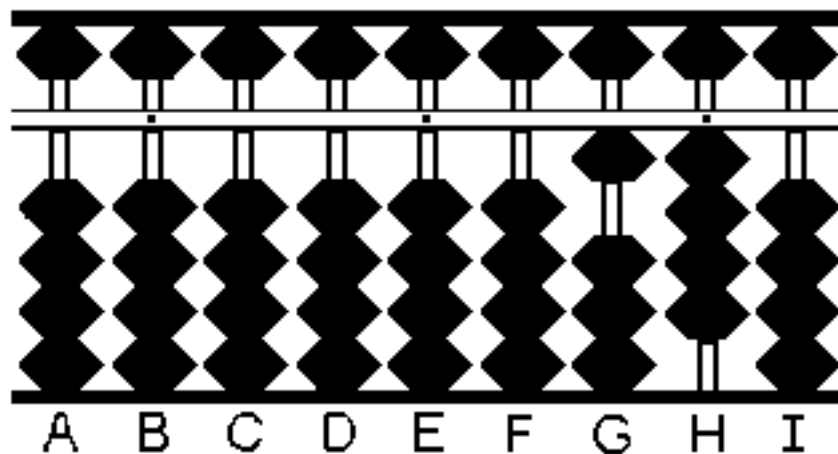
Now we want to add **1** to this. To do so would be simple enough if we had a single available earth bead on Rod H. But all of the earth beads already "have value" because they are pushed towards the reckoning bar. But we still have a heaven bead on rod H that does not "have value" because it is not pushed towards the reckoning bar. But it's value would 5 which is too large – so instead we "Add 5" (by moving the single heaven bead towards the reckoning bar) and "Subtract 4" to give us a net increase of just one. Now we have a frame that looks like:
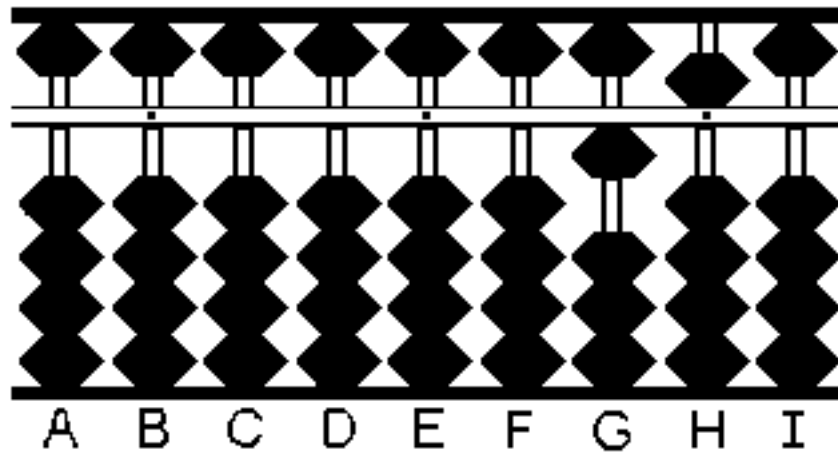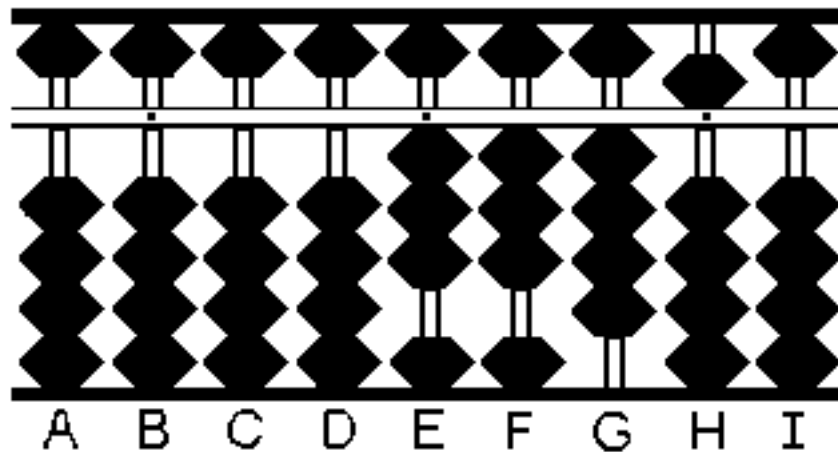
And you can see the result is **15** (14+1).

OK – now you are ready for something really meaty. Let's add some serious numbers – no matter how many numbers you deal with, just work with one rod at a time and generally proceed from left to right. Lets try 3345 + 6789 (=10134).

First, **Place 3345** on the frame:

Next (working from left to right) we want to "Add 6" of 6789 on rod E. This is simple enough – simply squeeze the one remaining earth bead and the heaven bead towards the reckoning bar which results in a frame that looks like this:
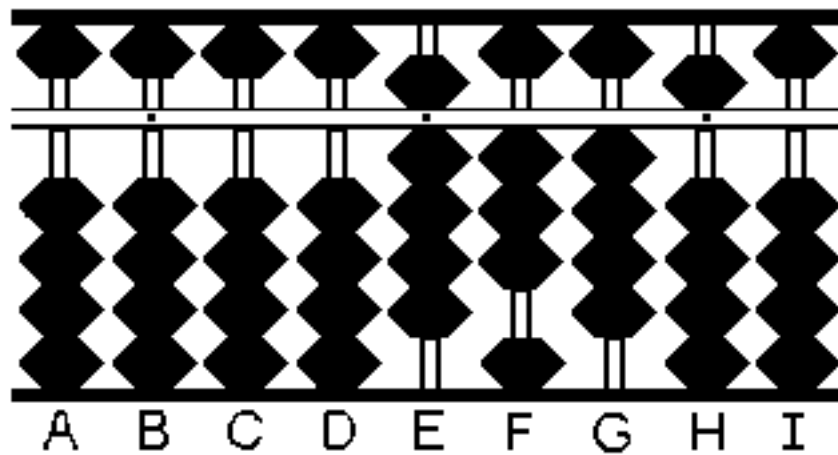
This is the number 9345 but we are not done – there are still three more numbers to go! Next we "Add 7" on rod F. Here we have to carry – a simple glance at the rod will show that there are not 7 worth of beads to move to the reckoning bar. So instead, we "Subtract 3, Add 10" (which is the same as adding 7). We do this by moving down 3 beads on Rod F ("Subtract 3") and then moving up a bead on Rod E (technically "Add 1" but this rod has 10x's the value of rod F so in effect it's like adding 10 to Rod F although the work is done with a single bead on the next column to the left – this is a key concept in the use of the Soroban!).

We would be done with this number except that there **are** no free beads on Rod E – so again we must carry to the next rod to the left and "Subtract 9, Add 10" which gives us the same effect. This leaves us with a single bead moved up on Rod D ("Add 1") and nothing on Rod E ("Subtract 9" which is the total of all the beads previously at the reckoning bar on this rod). This results in the following frame:
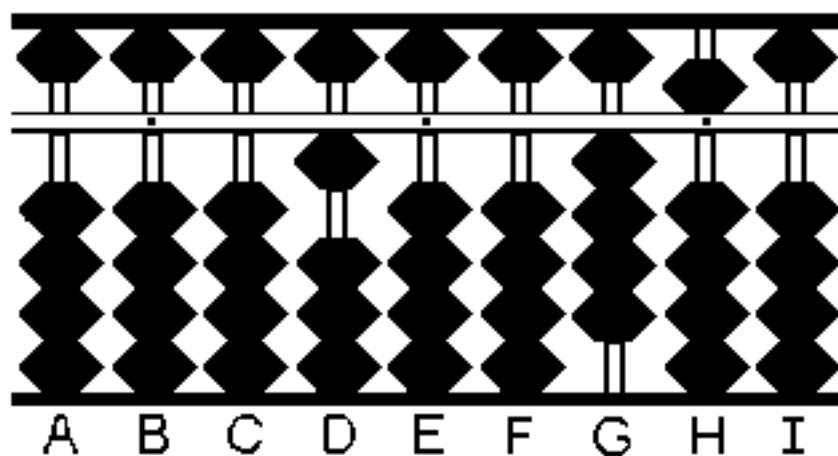

This is the number 10045. But again, we are not done. Just two more numbers to go. As you can see, the act of carry in addition to the next rod to the left is commonplace with addition – and with practice will become very easy – it can be done as quickly as someone can read off sets of numbers to you. I've found that with just a few weeks of practice, this process becomes very natural. Next we "Add 8" on rod G. Again we don't have enough beads to add 8 so we have to carry. In this case we "Subtract 2" and "Add 10". Therefore we move down two earth

beads from rod G and add a single bead to Rod F (which is effectively adding 10 to Rod G because it is one rod to the left). The resulting frame looks like:
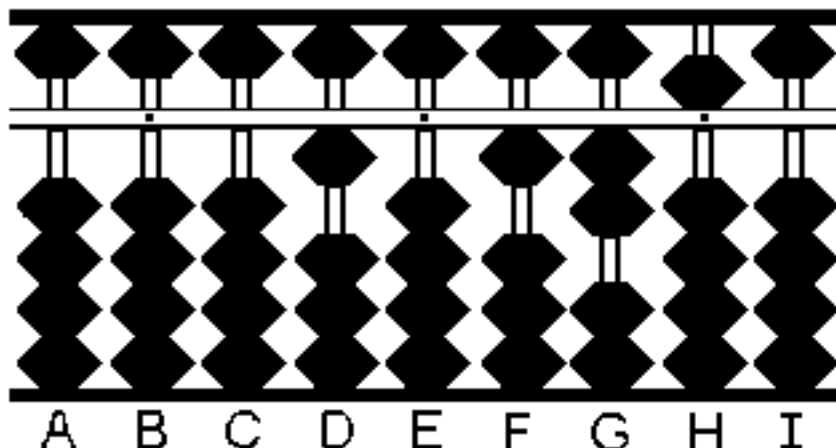
This is the number 10125. We are almost done! Just a final digit to add – the "9". So we "Add 9" to the unit rod H. Again, we don't have enough beads to do this with rod H so we must "Subtract 1" and "Add 10". To subtract 1, where we already have a value of 5, we must move away the heaven bead and move the four earth beads to the reckoning bar (essentially "Subtract 5", "Add 4" on this rod). Only after we deal with the unit rod H should we "Add 10" which is accomplished by going to Rod G and adding a single earth bead. When performing addition, always deal with the current rod first – and then if there is a carry move up a single bead on the rod to the left. These are very essential concepts when adding! After performing this step, we have the following resulting frame:
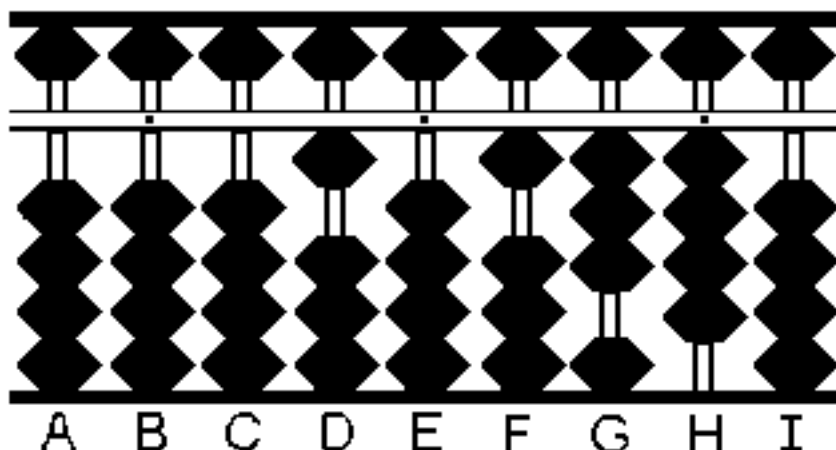

It's easy enough to read this frame and see that the resulting number is **10134** – which is the answer to our original problem (3345 + 6789)! A problem like this may seem strange or even awkward at first – but with a bit of practice, you should be able to work a problem like this in just a few seconds. Even faster and more accurately than you might be able to do it on paper!
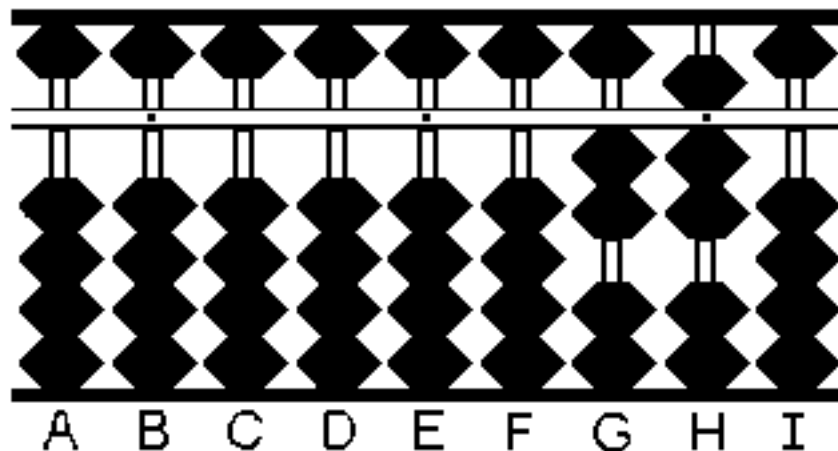
SUBTRACTION

Subtraction on the Abacus is almost as simple as Addition was - it's just the reverse process. Instead of having to deal with a possible carry in the 10's digit (next rod to the left), you now deal with a possible borrow on that same rod. In general, with subtraction you still work the problem and numbers from left to right - just deal with one rod at a time. If there are enough beads that "have value" on the current rod you can just subtract the desired number. If you don't have enough beads with value - you must first subtract 10 (by losing one beads worth of value on the rod to the left of the one you are working on) and then add on your current rod to make up the difference. We will give some concrete examples to show how easy this is.
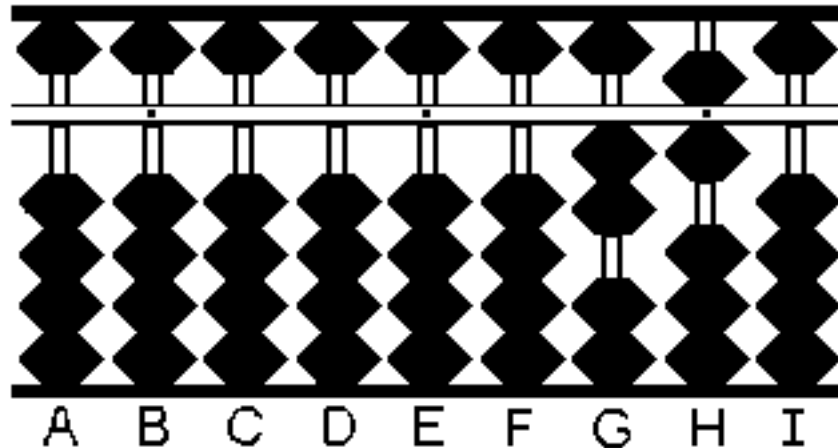
Let us take the number 47 (**Place 47**):
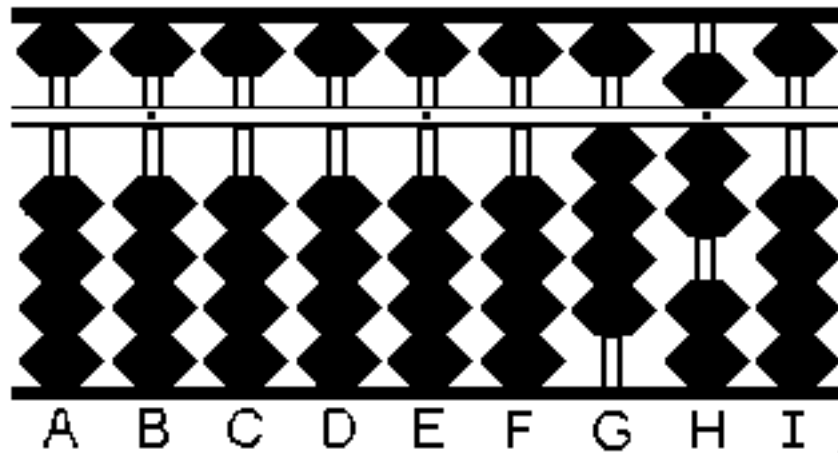
And let's subtract off 21. First we start on Rod G and "Subtract 2" which is easily accomplished by moving 2 earth beads away from the reckoning bar. We now have the value 27 (not the final answer yet):

Now we move on to the units rod H and "Subtract 1" by moving a single earth bead away from the reckoning bar. This yields us our final answer of 26:

Now let's subtract 4 from this. We can go right to the units rod and "Subtract 4" except that there are not enough single earth beads (worth 1 each) to subtract 4 so instead we must "Subtract 5, Add 1" to give us the same effect. Here we move a single heaven bead (worth 5) away from the reckoning bar and move a single earth bead (worth 1) up to the reckoning bar to accomplish this. We now have our final answer of 22 (which is the result of 26-4):
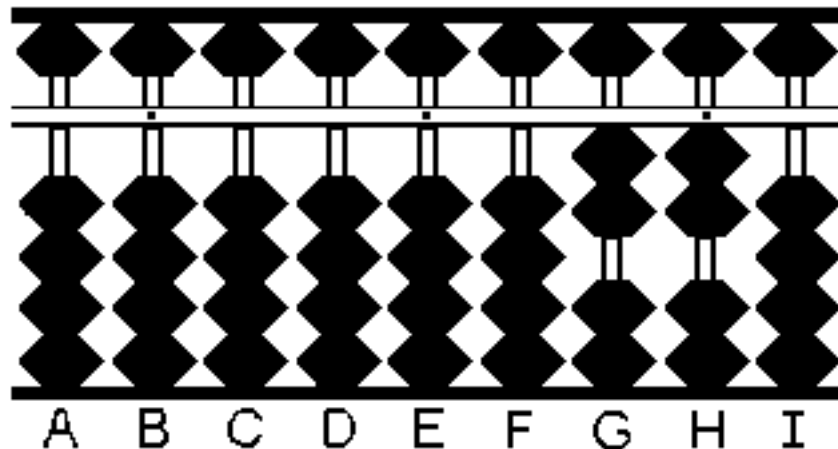
Now we show how to borrow. Let's take the 22 on the frame and subtract off 14. First we start with rod G and "Subtract 1" which is easy. Our result (not yet the final answer) is:
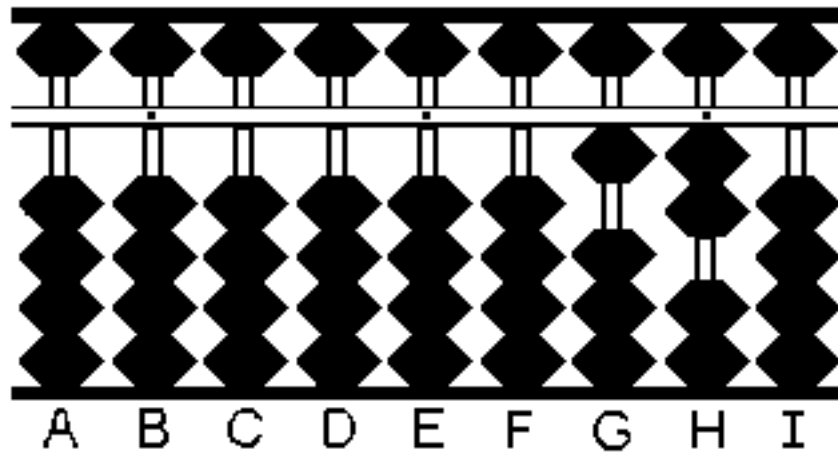
Now we must "Subtract 4" from our unit rod H. But we don't have 4 worth of beads to subtract on this rod - so instead we must "Subtract 10, Add 6" to produce the same result. To accomplish this, we subtract a single earth bead from rod G (effectively "Subtract 10" since rod G has 10 times the value with respect to rod H which we are working with) and then we "Add 6" to rod H (here we need to move both a Heaven bead and an Earth bead towards the reckoning to accomplish the "Add 6"). This results in a frame with **8** which is our answer!
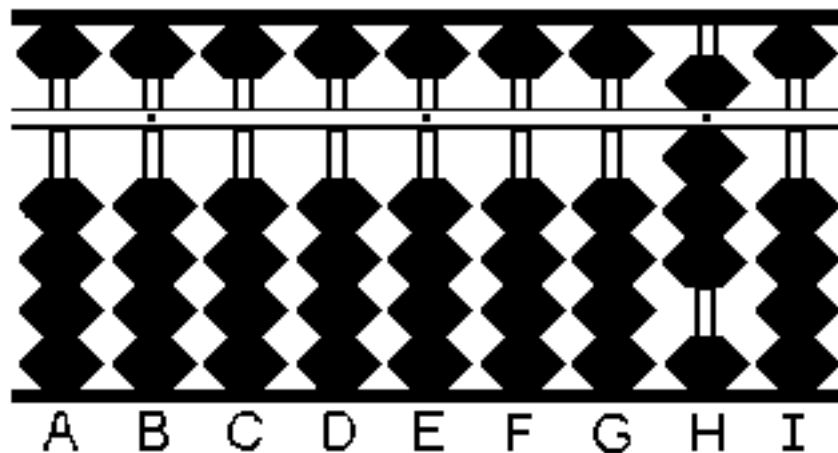
MULTIPLICATION

Multiplication is nothing more than a series of additions. However, it is not very convenient to do 23 separate adds on the number 47 just to give us the answer to 23x47 ! Therefore, there are specific techniques for performing multiplication on the abacus frame. I've learned two different methods and know there are probably others as well – but I will teach only the method that was approved by the Japan Abacus Committee. I have found that this method is least prone to errors and is very simple once you learn the basic technique. You will need to know

your multiplication table up to 9x9=81 and you should be all set.

Let's say you are given a problem like 23x47. The number 23 is called the multiplicand and the number 47 is called the multiplier. In general you place the multiplicand (here 23) near the center of the frame – and keep the last digit on a unit rod (marked with a dot) to help keep your place (especially important in multiplication). Then enter the multiplier (here 47) to the left of this number – skipping two clear rods going to the left (do not worry if this number falls properly on a unit dot – it's not necessary). Now you have both numbers on the frame. In the method I will describe you will be producing the answer just to the immediate right of the multiplicand (the number in the middle of your abacus frame). When we are done – the multiplicand will be gone and the answer will remain (along with the multiplier still to the left).

Lets place the 23 and 47 on the frame as we described above (except that I am only going to skip one blank column rod to conserve space in the diagrams! Some prefer skipping just one column and you might have to do it if you have a small abacus – but generally it is better to skip two full columns):
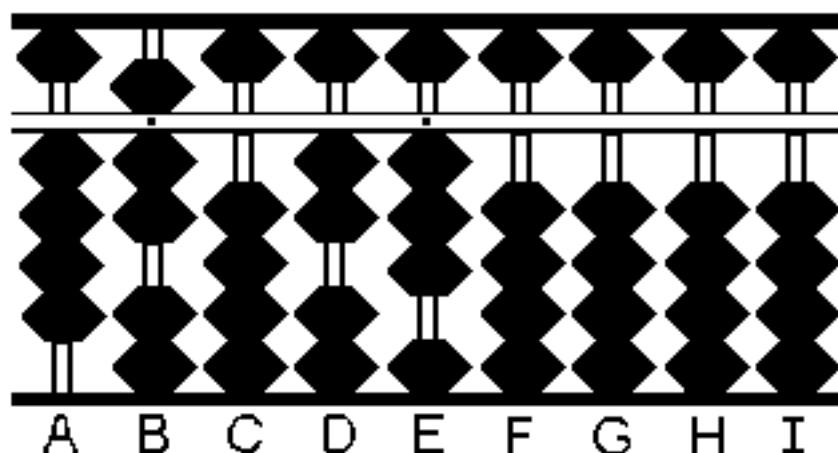
Now we are ready to multiply. It's similar to how you would work it on paper – except that the order of multiplication is a bit different (and should be followed exactly as I outline here).

First you work mostly with the multiplier. You take the right-most digit of the multiplier (in this case a '3') and you multiply it by the left-most digit of the multiplicand (in this case a '4'). The result is 12 – and so you add on the frame in the two rods to the right of the multiplicand (in this case FG). Since rods FG have no value, it's just a simple matter of "Add 1" to F and "Add 2" to G to produce the following frame:
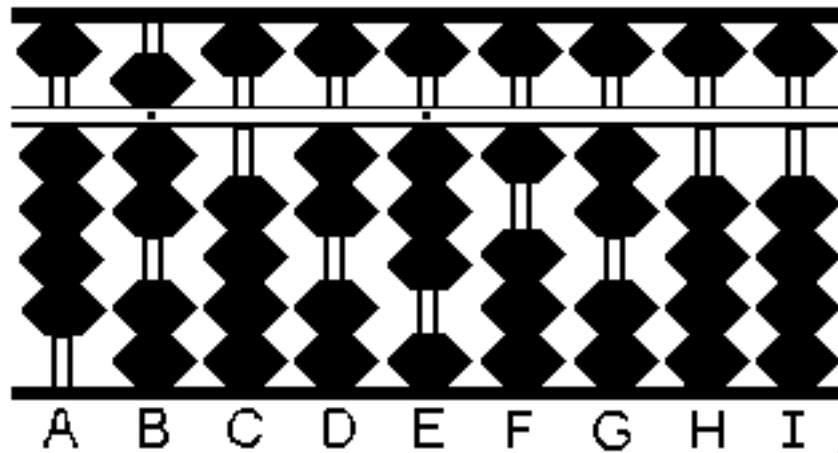
Now we multiply the same multiplier digit '3' against the next multiplicand digit '7' and put that result on GH (since we last left off at G with the last multiplication result). 3x7 is 21 and so we "Add 21" to rods GH which means we "Add 2" to G and "Add 1" to H. Simple enough and here is the resulting frame:
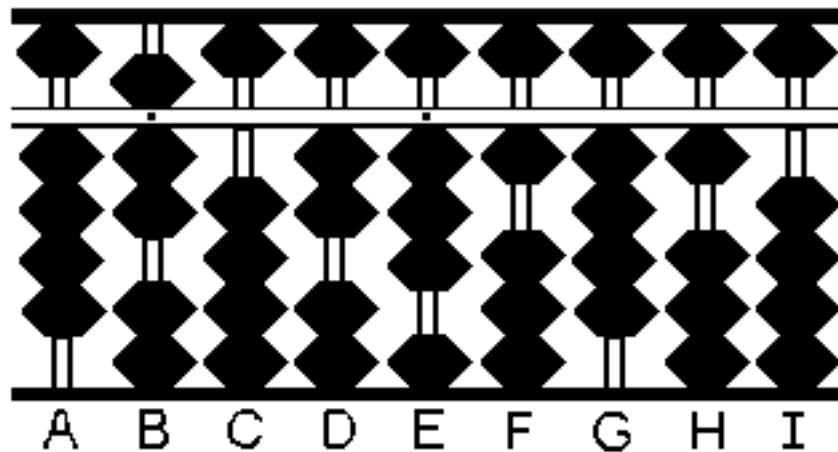
Now we are done with the first digit of the multiplier and so we must clear it from the frame – we simple clear the '3' off rod F. We will use this rod if there are more numbers in the multiplier left (and in this case there is another number left!).

Now we move on to the next digit of the multiplier – in this case the '2'. We multiply this times the left-most digit of the multiplicand (just as we did above for the first digit of the multiplier). And again we place the result (2x4 = 08) on the two rods next to this multiplier digit – rods EF. Important note – the result of 2x4 is a single digit 8 – but enter this as 08 so that you always use up 2 rods so you are technically doing: "Add 0" on E followed by "Add 8" on F. The resulting frame looks like:
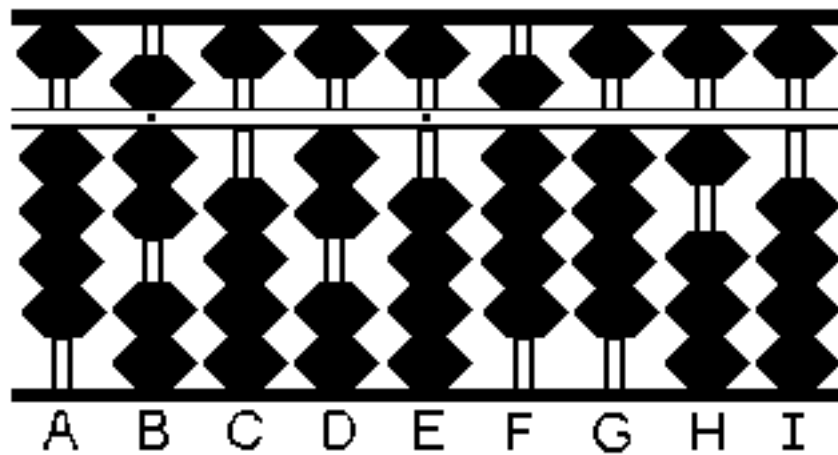
We are almost done – we just need to multiply the '2' times the remaining multiplicand digit of '7' and add the result of 14 to rods FG (remember we pick up where we left off so long as we haven't moved on to a new multiplier digit). In this case we "Add 1" to F which requires a carry to E (which you should already know how to do from the addition examples). And then you "Add 4" to G. You clear off the multiplier digit '2' which has now been processed and see that there are no more multiplier digits left (if there were, you just repeat the above processing). The resulting frame (after clearing away the '2' and you can also clear away the original '47' on the left hand side if you like – but we don't bother do it here) yields the resulting frame:
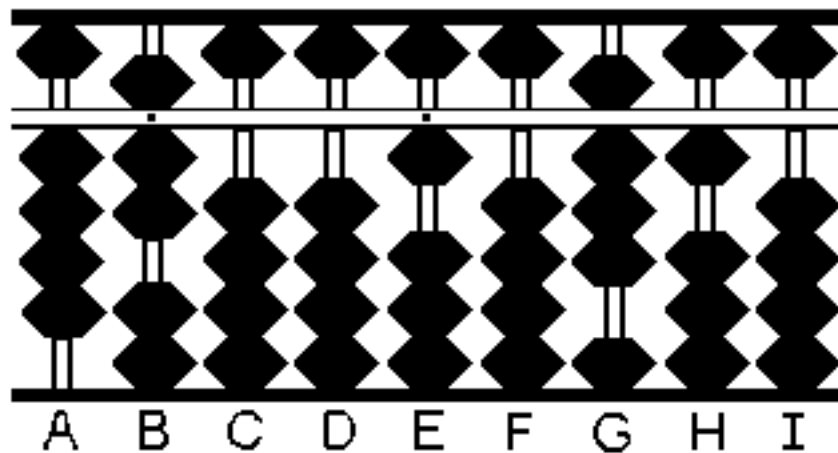
And so you can read the result of 23x47=**1081** although we still left the 47 on the left side of the frame (you can clear it – I tend to just leave it there). No matter how many digits you multiply, just apply the technique above and remember to work on the correct rod and it will go smoothly.

DIVISION (contributed by Totton Heffelfinger)

For me at least, division was a little intimidating. As it turns out Dave was right when he said, "Trust me. It's not that hard!" As with multiplication you will need to know your multiplication tables up to 9 x 9 = 81. It might be helpful to think of division as being nothing more than a series of subtractions

The techniques I use below are pretty much as they are described in "The Japanese Abacus - It's Use and Theory" by Takashi Kojima.

In describing the methods I will use standard terminology. For example in the problem $8 \div 2 = 4$, 8 is the dividend,   2 is the divisor, and 4 is the quotient.

Earlier in the handbook Dave wrote about the importance of the Unit rod. (Unit rods are those rods marked with a dot.) Unit rods seem particularly important in solving problems of division because the resulting quotient is often not a perfect whole number. In other words it forms a decimal. Because of this you should plan ahead and set up the problem on your abacus so that the unit number in the quotient falls on a unit rod. Also, in order to help you keep track of your calculations it is a good idea to set the unit number of your dividend on to a unit rod. As for the divisor, if it's a whole number it doesn't seem to matter much whether it follows any such rule.

Normally when setting up division problems on the abacus the dividend is set a little to the right of center and the divisor is set to the left. Traditionally the dividend and divisor are separated by three or four unused rods and this is where the quotient will be formed. Having said that, occasionally I find that four unused rods are not enough and I use more. It really depends on the problem.

Basically division is done by dividing a single digit into one or possibly two digits at a time. You're required to multiply after each division step and do subtraction to get the remainder. The remainder is then tacked on to the rest of the dividend and the division is continued in this way until its completed. It's not unlike doing it with pencil and paper but the abacus has the advantage of doing much of the grunt work for you.

As you will see I'll set up a few examples with both a whole number and a decimal in the quotient. All is better explained using examples. So let's get started.

Example 1. 837 ÷ 3 = ?

Step 1: Place the dividend 837on the right-hand side of the abacus (in this case on rods G, H & I) and the divisor 3 on the left (in this case rod B). Notice how the "7" in 837 falls on a unit rod.

This yields the resulting frame:
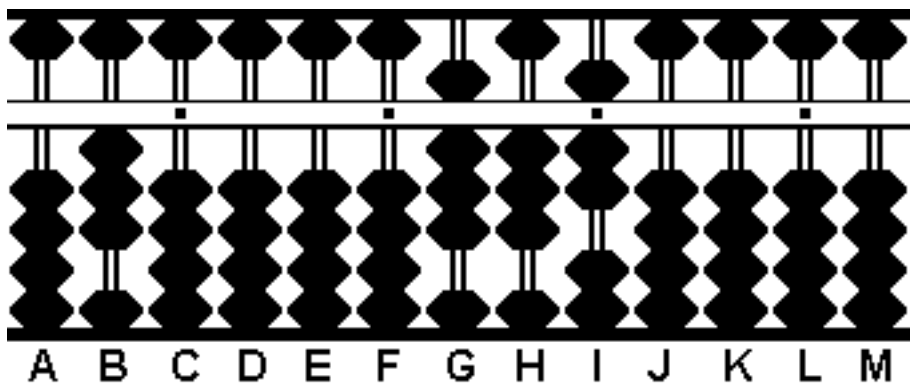

```
Step 1

A B C D E F G H I J K L M
          .       .       .       .
_____
0 3 0 0 0 0 8 3 7 0 0 0 0
```

Step 2: It looks like the quotient is going to have 3 whole numbers in its answer. Begin by placing the first number of the quotient on rod D, that way the unit number in the quotient will fall on unit rod F. In order to divide 837 by 3, start with the 8 in the dividend. It looks like 3 goes into 8 twice with a remainder. Place 2 on rod D. Multiply 2 x 3 to equal 6, then subtract 6 from 8 leaving a remainder of 2.

This yields the resulting frame:
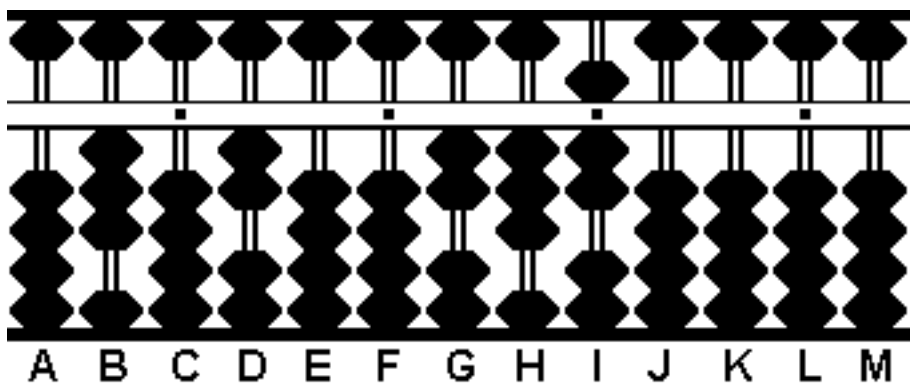

```
Step 2
A B C D E F G H I J K L M
          .       .       .       .
_____
0 3 0 0 0 0 8 3 7 0 0 0 0
      2
              – 6
_____
0 3 0 2 0 0 2 3 7 0 0 0 0
```

Step 3: Now there's 237 left with on rods G, H & I. Continue by dividing 23 by 3. It looks like 3 will go into 23 seven times with a remainder. Place 7 on rod E. Multiply 7 x 3 to equal 21, then subtract 21 from 23 leaving a remainder of 2.

This yields the resulting frame:
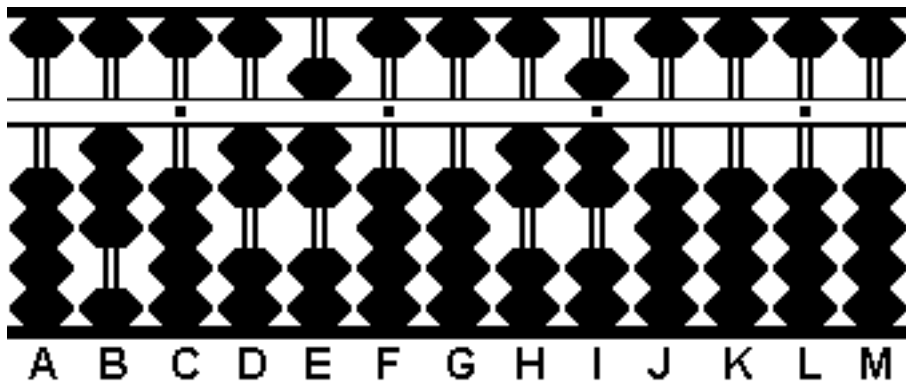
```
Step 3
A B C D E F G H I J K L M
          .       .       .       .
_____
0 3 0 2 0 0 2 3 7 0 0 0 0
              7
                  - 2 1
_____
0 3 0 2 7 0 0 2 7 0 0 0 0
```

Step 4 and Result: Now there's 27 left on rods H & I. Continue by dividing 3 into 27. 3 goes into 27 nine times perfectly. Place 9 on rod F. Multiply 9 x 3 which equals 27, then subtract 27 from 27 leaving 0. Notice how the "9" in 279 falls on unit rod F.

We're done. **279** is the correct answer
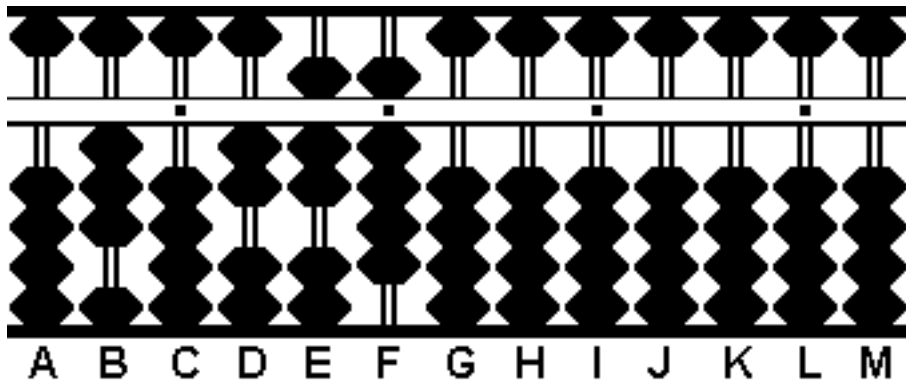

```
Step 4
A B C D E F G H I J K L M
          .       .       .       .
_____
0 3 0 2 7 0 0 2 7 0 0 0 0
              9
                  - 2 7
_____
0 3 0 2 7 9 0 0 0 0 0 0 0
```

Example 2. 6308 ÷ 83 = ?

Step 1: Place the dividend 6308 on the right-hand side of the abacus (in this case on rods F, G, H & I) and the divisor 83 on the left (in this case rods A & B). Notice how the "8" in 6308 is placed on a unit rod.

This yields the resulting frame:
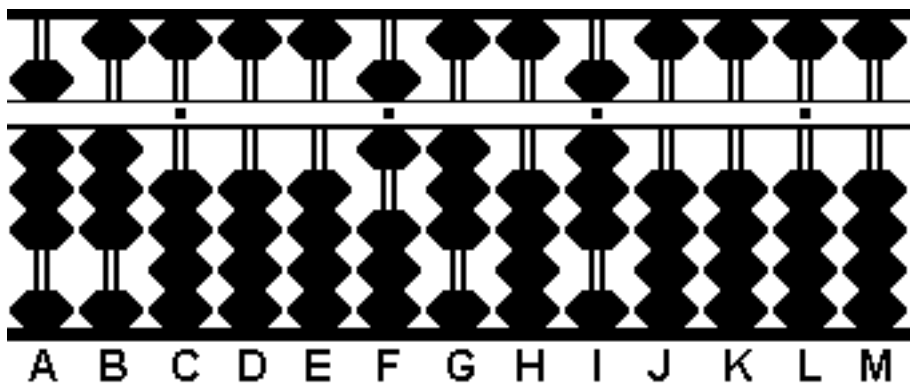

```
Step 1

A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
8  3  0  0  0  6  3  0  8  0  0  0  0
```

Step 2: In looking at the problem it's evident that 83 will not go into 6 nor will it go into 63. But it will go into 630. It looks like the quotient will have two whole numbers and a possible decimal. Therefore begin to form the quotient on rod E so that the unit number in the quotient will fall on unit rod F.

2a: In order to divide 6308 by 83, start with the 8 in the divisor and the 63 in the dividend. It looks like 8 goes into 63 seven times with a remainder. Place 7 on rod E. Multiply 7 x 8 which equals 56, and subtract 56 from 63 leaving a remainder of 7.

```
Step 2a

A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
8  3  0  0  0  6  3  0  8  0  0  0  0
            7
            -  5  6
8  3  0  0  7  0  7  0  8  0  0  0  0
```

2b: Now there's 708 left on rods G, H & I. Because we've multiplied the "8" in 83 by 7, we must multiply the "3" in 83 by 7. 7 x 3 equals 21. Subtract 21 from 70 leaving a remainder of 49.

The resulting frame after steps 2a & 2b:


```
Step 2b

A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
8  3  0  0  0  0  7  0  8  0  0  0  0
            7
               -  2  1
8  3  0  0  7  0  4  9  8  0  0  0  0
```
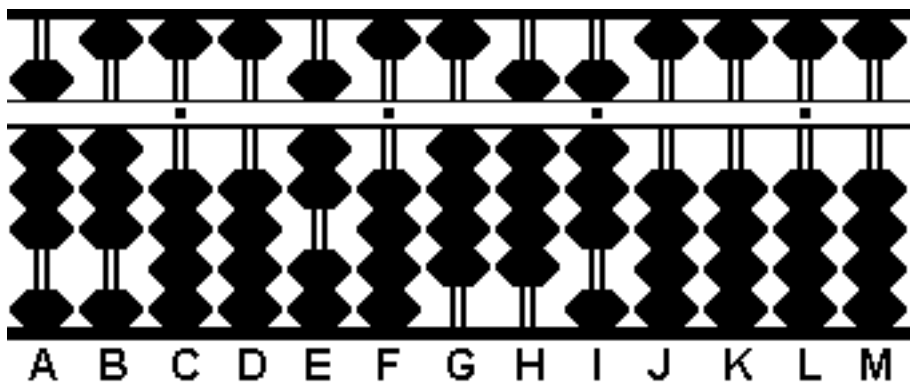
Step 3a: Now there's 498 left on rods G, H & I. Continue by dividing the 8 in the divisor into the 49 in the dividend. It looks like 8 goes into 49 six times with a remainder. Place 6 on rod F. Multiply 6 x 8 which equals 48. Subtract 48 from 49 leaving a remainder of 1.
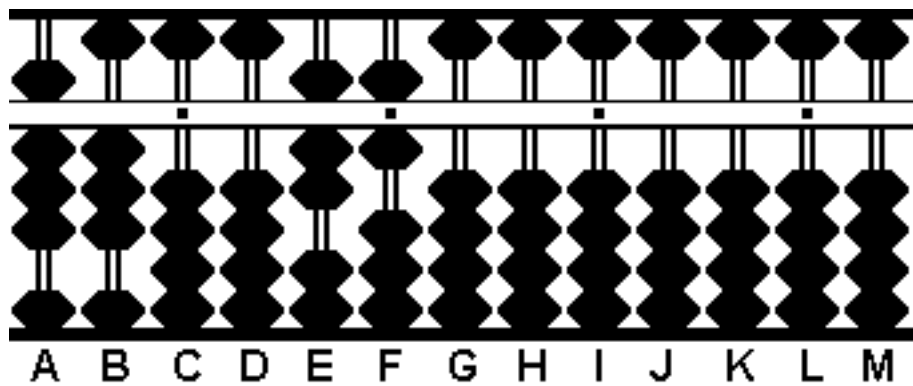
```
Step 3a
A B C D E F G H I J K L M
        .       .       .       .
8 3 0 0 7 0 4 9 8 0 0 0 0
              6
            − 4 8
8 3 0 0 7 6 0 1 8 0 0 0 0
```

3b: Now there's 18 left on rods H & I and we must multiply 6 by the 3 in the divisor. 6 x 3 equals 18. Perfect. Subtract 18 from 18 leaving 0. Notice how the "6" in 76 falls on unit rod F.

We're done. **76** is the correct answer.

```
Step 3b
A B C D E F G H I J K L M
        .       .       .       .
8 3 0 0 7 0 0 1 8 0 0 0 0
              6
              − 1 8
8 3 0 0 7 6 0 0 0 0 0 0 0
```
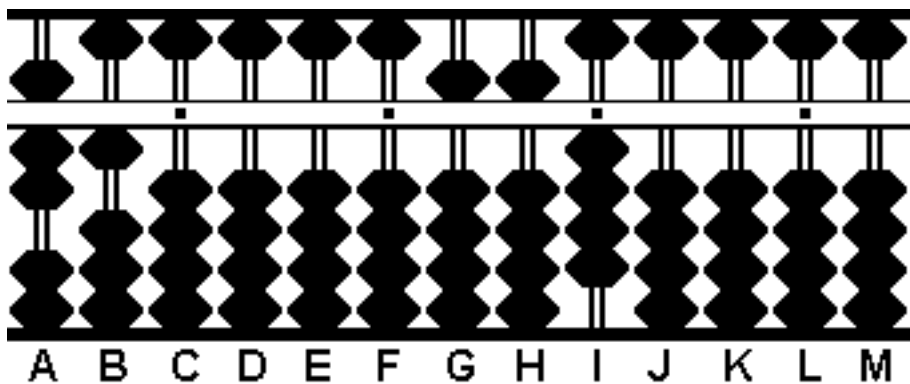
Example 3: 554 ÷ 71 = ?

Step 1: Place the dividend 554 on the right-hand side of the abacus (in this case on rods G, H & I) and the divisor 71 on the left (in this case rods A & B).

This yields the resulting frame:

Step 1

A	B	C	D	E	F	G	H	I	J	K	L	M
			.			.			.			.
7	1	0	0	0	0	5	5	4	0	0	0	0

Step 2: In looking at the problem it's evident that 71 will not go into 5 nor will it go into 55. But it will go into 554. It looks like the quotient will have one whole number and a decimal. Therefore begin to form the quotient on rod F. Anything after that will be a decimal.

2a: In order to divide 554 by 71, start with the 7 in the divisor and the 55 in the dividend. It looks like 7 goes into 55 seven times with a remainder. Place 7 on rod F. Multiply 7 x 7 which equals 49, and subtract 49 from 55 leaving a remainder of 6.

Step 2a

A	B	C	D	E	F	G	H	I	J	K	L	M
			.			.			.			.
7	1	0	0	0	0	5	5	4	0	0	0	0
					7							
						–	4	9				
7	1	0	0	0	7	0	6	4	0	0	0	0

2b: Now there's 64 left on rods H & I and we must multiply 7 times the 1 in the divisor. 7 x 1 equals 7. Subtract 7 from 64 leaving 57.

The resulting frame after steps 2a & 2b:
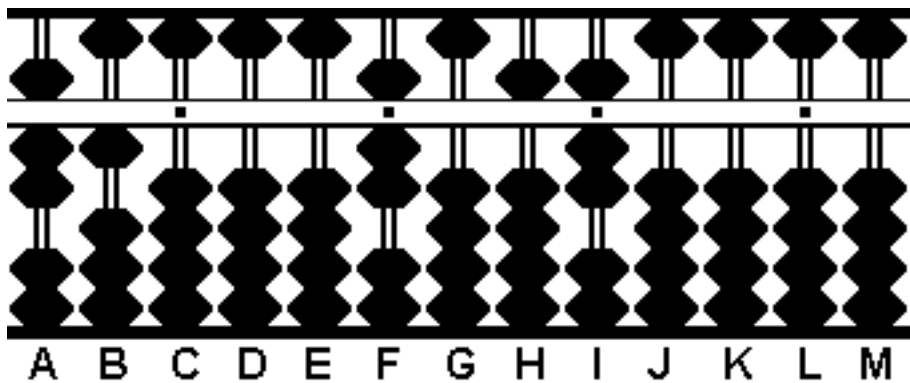
Step 2b

A	B	C	D	E	F	G	H	I	J	K	L	M
			.			.			.			.
7	1	0	7	0	0	0	6	4	0	0	0	0
							7					
								–	7			
7	1	0	0	0	7	0	5	7	0	0	0	0

Step 3: Now we're in decimal territory. If we want to continue we're going to have to take a zero from rod J. Let's continue.

3a: Now that we've decided to use a decimal we've got 570 on rods H, I & J. Continue by dividing the 7 in the divisor into the 57 in the dividend .It looks like 7 goes into 57 eight times with a remainder. Place 8 on rod G. Multiply 8 x 7 which equals 56, and subtract 56 from 57 leaving a remainder of 1.

```
Step 3a
A B C D E F G H I J K L M
        .       .       .       .
7 1 0 0 0 7 0 5 7 0 0 0 0
              8
                  –  5 6
7 1 0 0 0 7 8 0 1 0 0 0 0
```

3b: We've got 10 left on rods I & J and we must multiply 8 by the 1 in the divisor. 8 x 1 equals 8. Subtract 8 from 10 leaving 2. (Notice how the 8 in the quotient has fallen on the first decimal rod G.)

The resulting frame after steps 3a & 3b:
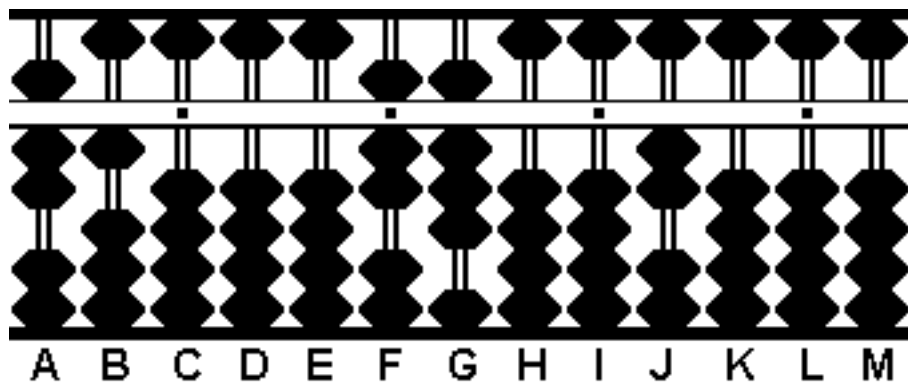

```
Step 3b
A B C D E F G H I J K L M
        .       .       .       .
7 1 0 0 0 7 0 0 1 0 0 0 0
              8
                      –  8
7 1 0 0 0 7 8 0 0 2 0 0 0
```
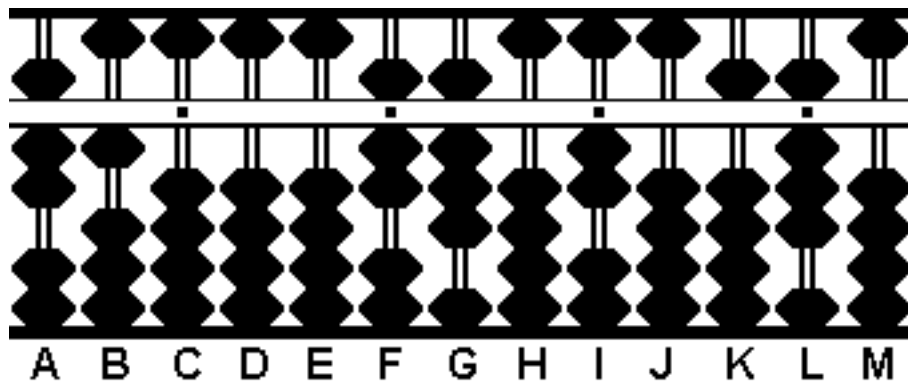
Step 4: Okay. Now there's 2 left on rod J. If we're going to continue we're going to have to take another zero, this time from rod K. That gives us a total of 20. 20 cannot be divided by 71. We must take another zero from rod L. (Note the quotient will have zero on rod H.)

4a: Now there's 200 on rods J, K & L. 200 is divisible by 71. Continue by dividing the 7 in the divisor into the 20 in the dividend. It looks like 7 will go into 20 two times with a remainder. Place 2 on rod I. Multiply 2 x 7 which equals 14. Subtract 14 from 20. That leaves 6.

Step 4a

A	B	C	D	E	F	G	H	I	J	K	L	M
		.			.			.			.	
7	1	0	0	0	7	8	0	0	2	0	0	0
									2			
								–	1	4		
7	1	0	0	0	7	8	0	2	0	6	0	0

4b: We've got 60 on rods K & L. Now we have to multiply 2 by the 1 in the divisor. 2 x 1 equals 2. Subtract 2 from 60 leaving 58.

The resulting frame after steps 4a & 4b:

Step 4b

A	B	C	D	E	F	G	H	I	J	K	L	M
		.			.			.			.	
7	1	0	0	0	7	8	0	0	0	6	0	0
									2			
										–	2	
7	1	0	0	0	7	8	0	2	0	5	8	0

Step 5: Now we've got 58 left on rods K & L. In order to continue we're going to have to take another zero, this time from rod M. (Because we're running out of rods, this will be the final step.)

5a: Now there's 580 on rods K, L & M. Continue by dividing the 7 in the divisor into 58 in the dividend. It looks like 7 will go into 58 eight times with a remainder. Place 8 on rod J. Multiply 8 x 7 which equals 56. Subtract 56 from 58. That leaves 2.

Step 5a

A	B	C	D	E	F	G	H	I	J	K	L	M
		.			.			.			.	
7	1	0	0	0	7	8	0	2	0	5	8	0
									8			
										–	5	6
7	1	0	0	0	7	8	0	2	8	0	2	0

5b: We've got 20 on rods L & M. Now we have to multiply 8 times the 1 in the divisor. 8 x 1 equals 8. Then subtract 8 from 20 which equals 12. Even though this division problem could go further, this will be as far as we can go because we've run out of room. As with any calculator, the abacus has a limit to how many numbers it can accommodate.

We've arrived at 7.8028 as shown on rods F through J. (The remainder 12 on rods L & M can be ignored.) Rounding off to three decimal points, the answer is **7.803**.
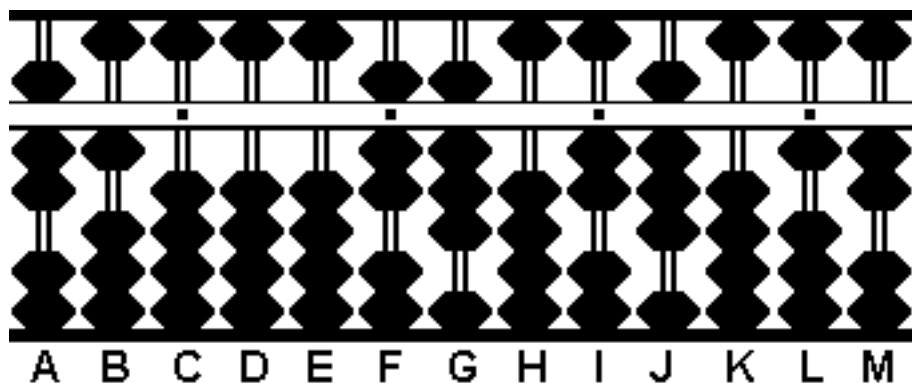
Step 5b

A	B	C	D	E	F	G	H	I	J	K	L	M
			.			.			.			.
7	1	0	0	0	7	8	0	2	8	0	2	0
									8			
											–	8
7	1	0	0	0	7	8	0	2	8	0	1	2

That's basically all there is to it. No matter how many digits you find in your divisor or in your dividend, just continue dividing using the above method. For me I find the best way to learn is to use my electronic calculator. (Blasphemous, I know.) What I do is I punch in 3 or 4 random numbers and divide them by 2 or 3 other random numbers. Then without looking at the answer, do the work on my abacus. Then I use the calculator to check my work. My abacus is always way more fun.

⬡ ADVANCED OPERATIONS

- *Extracting Square Roots*
- *Use of Decimals for Multiplication and Division*
- *Negative Numbers*
- *Many Additional Advanced Techniques for the Soroban and Suan Pan*

⬡ ADDITIONAL CONSIDERATIONS

Daily Practice

The use of the Soroban/Abacus is best learned through practice. Find sets of numbers you can add and subtract - perhaps numbers in a math textbook. Or add successive numbers in the phone book. Or use a computer program to generate lots of random numbers to add.

Whichever way you do it - a bit of daily practice will soon make you very comfortable in the use of the abacus. One proven technique for learning addition and subtraction is to take the number "**123456789**" and add this **nine** (9) times. You will get a very neat looking resulting answer (if you did it right!) of "**1111111101**". If you then subtract from this resulting answer "123456789" nine times you will get zero again. Adding and subtracting this special number nine times will teach you nearly every basic manipulation you will ever encounter for addition and subtraction and will improve your speed and accuracy. When I first started it took me about 4 or 5 minutes to add "123456789" nine times and I was only right about half the time. Now I can do it in about 2 minutes and am right about 80% of the time. I continue to practice this and get better and I recommend it as a good "warm-up" exercise when you are using the abacus.

Japanese Soroban Grades/Rankings

The Japanese actually take the use of the Soroban fairly seriously. I'm sure it's gone into a bit of a decline in recent years, but I've read that they teach their children the use of Soroban in the early elementary years and some schools here in the US have given classes on the Soroban and found that they generally improve the math skills of children who learn it. The Japanese also have a ranking system - starting at level 6 (or grade 6) which is somewhat skilled and moving down to level 5 (more skilled) all the way down to level 1 (Soroban mastery). They say that a gifted child who studies the Soroban from an early age can attain a level 1 ranking - while someone who takes up the study of this instrument after they are grown will only ever get to the 3rd or maybe 2nd level. At a level 6 – you must be able to accurately add and subtract sets of 15 different numbers of size 3 or 4 digits. Also, multiplication and division of 2 and 3 digits numbers must be demonstrated. All of this requires a high level of accuracy and each set of numbers is timed. By the time you reach level 3 the numbers have become very large and varied – often mixing 2 and 3 digit numbers with 6 and 7 digit numbers in addition and subtraction. Multiplication and division are now using very large numbers – often 4 or 5 digits and they must also deal with decimal places! Time constraints are much more strict as well! I will try to provide some level 6 practice exams in this handbook in the future for those of you who want to gauge your progress.

 # RESOURCES

A Where can I buy a Soroban/Abacus?

A Japanese Soroban and/or a Chinese Abacus are not easy to come by locally. There is only one company that I know of that still manufactures new Sorobans – and that would be the Tomoe Soroban Company in Japan (www.soroban.com) . They will sell you a beautiful Soroban – however you need to fax in your order which can be a bit of a pain. Hopefully soon

they will have secure web ordering!

News! I just found another website that lists Japanese Sorobans for sale - http://www.citivu.com/usa/sigmaed/ seems to have several of the same makes and models as the Tomoe Soroban Company – and they are based here in the US (California).

As for a Chinese abacus, if you live in a big city which has a Chinatown, you might be able to find a store that sells one of these instruments. Otherwise there are some places online you can check (www.asianideas.com and www.tigergifts.com/abacus.htm and www.onlineoriental.com)

But possibly the best way to pick up a quality Abacus or Soroban is to check the online auction sites such as Ebay (www.ebay.com). I don't want to go into detail on how these auction sites work – and I'll leave it up to you to determine if this route is acceptable. But I can tell you that every week someone seems to be selling an Abacus or Soroban – and often they go very cheaply. I picked up my first 4 Sorobans (3 wooden, 1 plastic all different sizes) for about US$10 each (the plastic which is one of my favorites because of it's size and durability only cost me US$2). A good 13 or 17 column Soroban should be easy to get in the US$10 range – and a stunning vintage 21 or 23 column wooden Soroban should be less than US$20. When you search, be sure to search for both Abacus and Soroban – sometimes the seller does not know which he/she has – and often good Japanese Sorobans are listed under the title of "abacus".

Recommendations for purchasing a Soroban/Abacus.

For a traditional Chinese abacus, I recommend a basic 13 rod frame which is about 12x7 (stay away from the little tiny palm sized ones 2x3 inches). For a Soroban, I recommend a wood frame instrument that is traditional size. For the Japanese Soroban the 13, 17 and 21 are popular sizes (number of rods). The more rods, the larger the numbers it can handle (this is mostly important for multiplication and division – for addition and subtraction smaller instruments would be very adequate). For average use, a 13 column instrument is fine – although a 17 or 21 will allow you do perform nearly any sized calculation. A typical 13 column Soroban is about 7 inches long and about 2.5 inches tall. A big 27 column instrument is about 15 inches long and 2.5 inches tall.

Recommendations for Books about Soroban/Abacus.

There are three books that I have read on the subject (books of this sort are not easy to come by!). The first is still in print – it's called "The Abacus" by Jesse Dilson and comes with a mini wooden Chinese abacus so you can actually use it while learning! The problem is that the book is not the best – some of the techniques are inefficient and the method of multiplication is not as good as the Japanese method. Also, the book is devoted to the Chinese abacus – and has

virtually no information on the Japanese Soroban (of course you can easily work one from learning the other). But it is still a good source of information and worth a look.

The second was the The Japanese Abacus Explained by Yoshino, Y. The Yoshino book is fairly good - it covers all the basics of bead manipulation with diagrams (all the older Japanese style with 1 bead on top, 5 on the bottom) and has lots of exercises for addition, subtraction, multiplication and division. Some notes on dictating numbers to someone to add is also given. But overall the book is not as good nor as well written as the Kojima book (see below).

The best book on the use of the Abacus is "The Japanese Abacus – It's Use and Theory" by Takashi Kojima. An all English book published by Charles E. Tuttle company. The book is hard to come by – but the publisher still had some copies in stock last time I checked their website (do a search for Charles E. Tuttle) and I picked up my copy used for about US$10. It's just over 100 pages and absolutely packed with good advice and all practical information on mastering the Abacus (technically the book uses illustrations from a Soroban just the same as my handbook). This book is most excellent – and easy read and very informative. I'd say it's probably the definitive work on the subject!

Recommended Websites.

Tomoe Soroban Company: http://www.soroban.com

The Art of Calculating with the Beads: http://www.ee.ryerson.ca:8080/~elf/abacus/

Soroban from The League of Japan Abacus Associations
http://www.syuzan.net/english/index.html

Discover Abacus: http://www2.ittu.edu.tm/math/ttm-math/mathematics/mat%20fuar/abacus/soroban/

The Abacus: A History: http://fenris.net/~lizyoung/abacus.html

A "Java" Abacus for your use: http://www.mandarintools.com/abacus.html

SOROBAN DISCUSSION GROUP!

I have started a Soroban / Abacus discussion group which is a free email group for the purpose of sparking some discussion on the use and theory of these fascinating calculating tools. You

can see the Soroban / Abacus discussion homepage at:

http://groups.yahoo.com/group/SorobanAbacus

Or email me at daveber@gis.net for more details -- I can get you signed up if you are interested – or you can sign up yourself at:

Subscribe to SorobanAbacus

Powered by groups.yahoo.com

⬣ SQUARE ROOTS (contributed by Totton Heffelfinger)

When it comes to extracting square roots, it's true; I can't remember the last time I needed to extract the square root of a large number and apply it to anything practical. I guess I just wanted to learn the technique because I think it's so amazing that it can be done on the abacus. In fact not only can it be done, using an abacus makes it easy.

There are different techniques that can be used to extract square roots using an abacus. I'll use the one that Takashi Kojima seems to favor in his book, *Advanced Abacus - Theory and Practice*. Rather than get muddled up with a bunch of algebraic formulas, I'll stick to how it's done on the abacus. For me this makes it more interesting. It certainly makes it less confusing because I don't understand algebraic formulas anyway.

In order to illustrate the technique, I'll use the terminology **square number**, **root number** and **square**. The **square number** refers to the number from which the square root is to be extracted. It's always set on the right-hand side of the abacus. The **root number** refers to the number that eventually forms our answer. It's always set on the left-hand side of the abacus. To **square** (or squaring) a number refers to multiplying a number by itself as in the example, 3 x 3 = 9.

In preparing to find the square root of a number, whether it be on paper or on the abacus, the number is normally separated into pairs of digits. In the example 17,362**.**236 starting at the decimal point and moving left and right the numbers would be separated 1' 73' 62'**.** 23' 60. Because of the constraints of using a small cyber abacus, we will do the separation and pairing mentally. Kojima does the same in his book.

Although the technique involves only simple mathematics and is therefore quite easy to do, the extraction process is fairly involved and labor intensive. For this reason trying to explain how it all works may be an exercise in confusion. It might just be better to jump ahead and learn from the examples below. However, by way of a brief explanation:

- Begin by placing the number from which the square root is to be extracted onto the right hand side of the abacus. This number on the right is the "square number".

- Group the square number into pairs as described above.

- Look at the first group of paired numbers (or number) and determine the largest perfect square that is less than or equal to the group. Subtract the perfect square from the group.

- Dealing again with the perfect square, take its square root and place it on the left hand side of the abacus. This number on the left is the "root number" and will eventually form our answer.

- Double the root number and divide it into the next group in the square number on the right hand side.

Place the quotient along-side the first root number.

- Mentally square the quotient and subtract it from the square number on the right hand side.

- Continue the process until the square number on the right is used up. In the event the square number is not a perfect square, when the decimal is reached further groups of 00 may be added and the process continued.

- When finished go back and halve the numbers that you doubled in the root number on the left. What remains will be the square root answer.

Confused? I think I would be. But take heart. Doing the following examples below on your abacus should clarify things.

Example 1. √ 1156 = ?

Step 1: Set 1156 on the right hand side of your abacus. This is the square number. If there's room, pair it off 11   56 or do it mentally. Because there are two groups (11 & 56) the answer will have two whole numbers possibly followed by a decimal in the answer.

This yields the resulting frame:
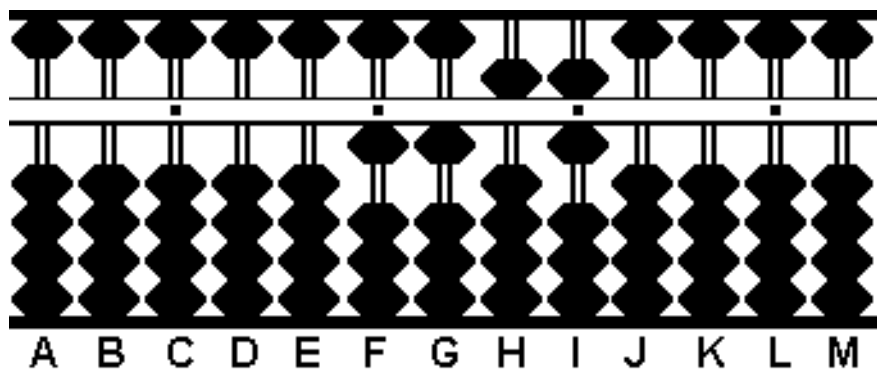

```
Step 1
A  B  C  D  E  F  G  H  I  J  K  L  M
       .        .        .        .
0  0  0  0  0  1  1  5  6  0  0  0  0
```

Step 2: Now look at the first group of numbers. (In this case the 11 on rods F & G.) Find the largest perfect square that is less than or equal to 11. The answer is 9. Subtract 9 from 11 leaving 2. Set the square root of 9 (namely 3) somewhere on the left-hand side of the abacus, in this case on rod B. This forms the first part of the root number.

Now the frame looks like:
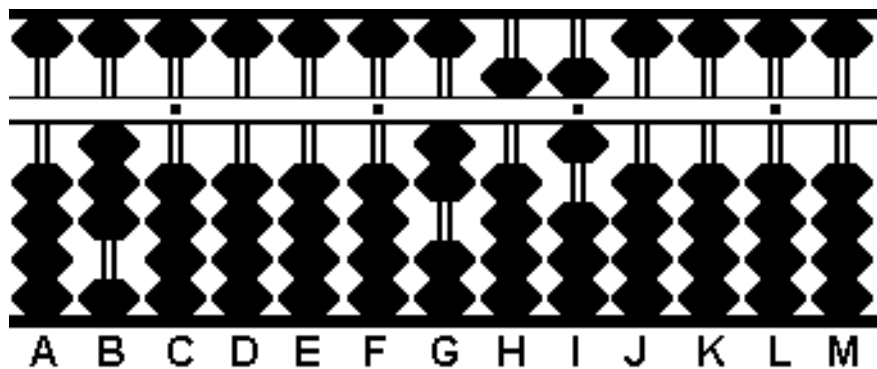

```
Step 2
A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
0  3  0  0  0  1  1  5  6  0  0  0  0
                  -  9
0  3  0  0  0  0  2  5  6  0  0  0  0
```

Step 3: Double the 3 on rod B to equal 6 and divide 6 into 25 on rods G & H. 6 goes into 25 four times with a remainder. Place 4 on rod C. Multiply 6 x 4 to equal 24 and subtract 24 from 25. That leaves 16 on rods H & I.
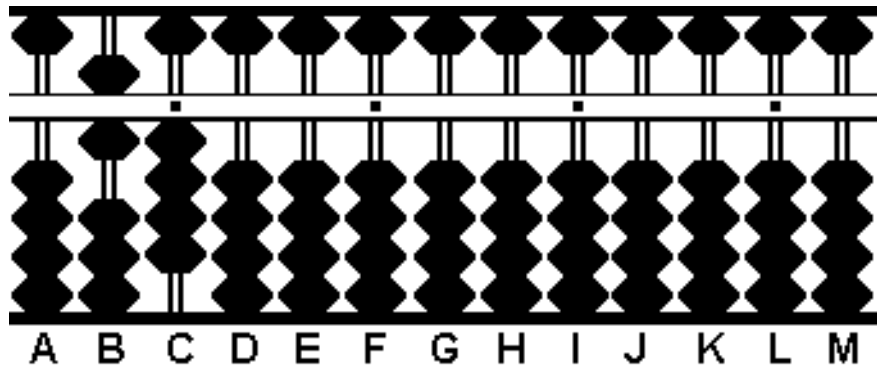
The result:
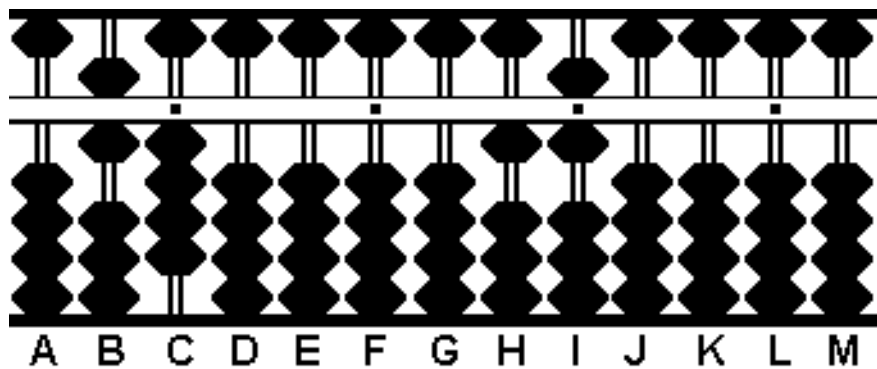

```
Step 3
A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
0  6  0  0  0  0  2  5  6  0  0  0  0
      4
                     -2  4
0  6  4  0  0  0  0  1  6  0  0  0  0
```

Step 4: Now mentally square the 4 on rod C and subtract the product from the remainder on the right. 4 x 4 = 16. Perfect. Subtract 16 from rods H & I.

This yields the resulting frame:


```
Step 4
A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
0  6  4  0  0  0  0  1  6  0  0  0  0
       (4)
                     -  1  6
0  6  4  0  0  0  0  0  0  0  0  0  0
```

Step 5: Almost done. In Step 3 we doubled the 3 on rod B to equal 6. Now it must be halved to its original 3

and we have the answer to the problem on rods B & C.

The answer **34** on rods B & C.
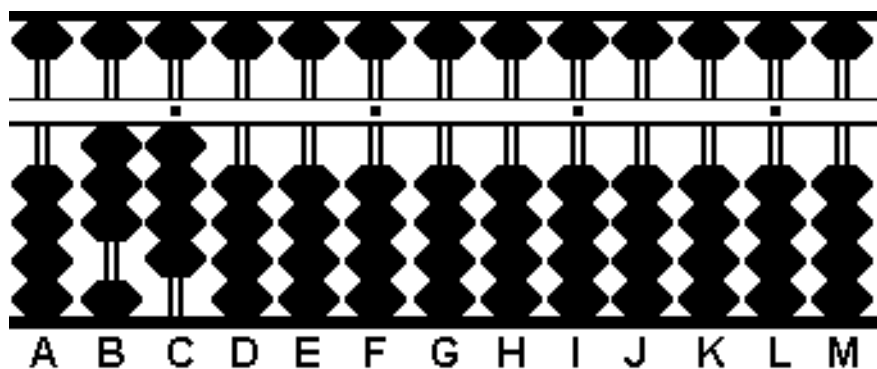
$$\sqrt{1156} = 34$$

* I've included this next example because it illustrates where it's possible to get a little confused and take a wrong turn. While the problem is easily overcome, it is something to watch for.

Example 2. √ 74,529 = ?

Step 1: Set 74,529 on the right hand side of your abacus. This is the square number. If there's room, pair it off 7   45   29 or do it mentally.   **Notice the odd pairing. The number 7 stands alone and will be considered the first group.**   Because there are three groups (7, 45 & 29) we know the answer will have three whole numbers possibly followed by a decimal.

The frame now:
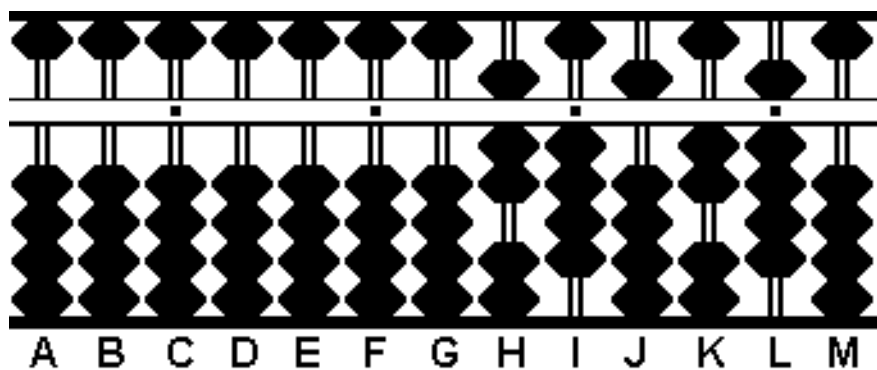

```
Step 1
A  B  C  D  E  F  G  H  I  J  K  L  M
        .        .        .        .
0  0  0  0  0  0  0  7  4  5  2  9  0
```

Step 2: Look at the first group of numbers. (In this case there's only the 7 on rod H.) Find the largest perfect square that is less than or equal to 7. The answer is 4. Subtract 4 from 7 leaving 3. Set the square root of 4 (namely 2) somewhere on the left-hand side of the abacus, in this case on rod D. This forms the first part of the root number.

The result:
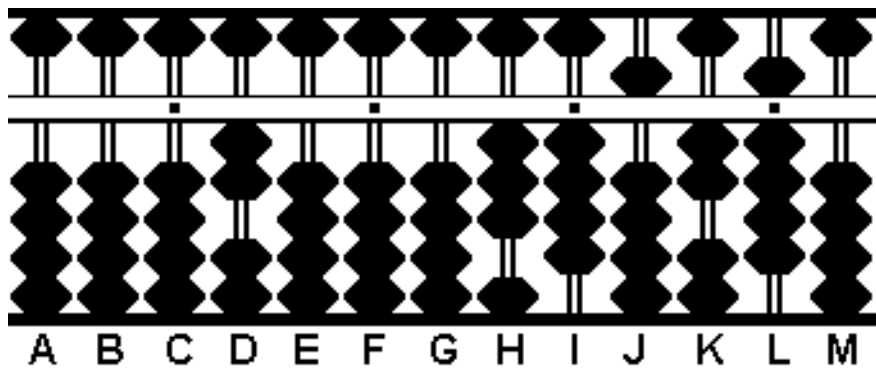

```
Step2
A B C D E F G H I J K L M
_____
        .       .       .       .
0 0 0 2 0 0 0 7 4 5 2 9 0
                  – 4
_____
0 0 0 2 0 0 0 3 4 5 2 9 0
```

Step 3: Now double the 2 on rod D to equal 4 and divide 4 into 34 on rods G & H. Convention has it that 34 ÷ 4 = 8 with a remainder. But....

> * This is where it gets interesting. The technique requires that after dividing 4 into 34 you <u>have</u> to subtract the square of the quotient. In other words, you have to have enough left over in the remainder. See it? If you choose 34 ÷ 4 = 8 there will only be 25 remaining on rods I & J. That's not enough left over to subtract 8 x 8 = 64. However, if you choose 34 ÷ 4 = 7 there will be 65 remaining on rods I & J. And 65 leaves you enough left over to subtract the square 7 x 7 = 49. See what I mean? Even though it's the correct answer to the division problem, you can't use 8 because you can't subtract its square. You have to go with 7. While this kind of problem doesn't always come up, it's something that can happen.

Okay, 7 it is. 4 goes into 34 seven times with a remainder. Place 7 on rod E. Multiply 4 x 7 to equal 28 and subtract 28 from 34 on rods H & I. With 65 left over on I & J there's enough to work with in Step 4.

This yields the resulting frame:
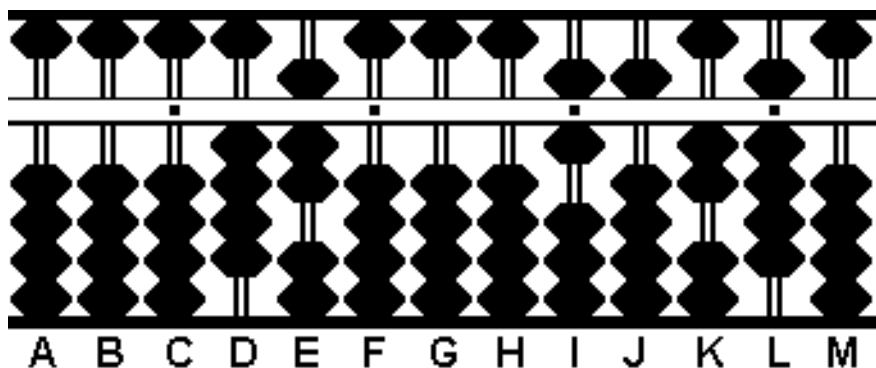

```
Step 3
A B C D E F G H I J K L M
_____
        .       .       .       .
0 0 0 4 0 0 0 3 4 5 2 9 0
        7
                  – 2 8
_____
0 0 0 4 7 0 0 0 6 5 2 9 0
```

Step 4: Now that we've ensured there's enough left over to work with, take the 7 on rod E and subtract its square from the remainder on the right. 7 x 7 = 49. Subtract 49 from 65 on rods I & J leaving 16.
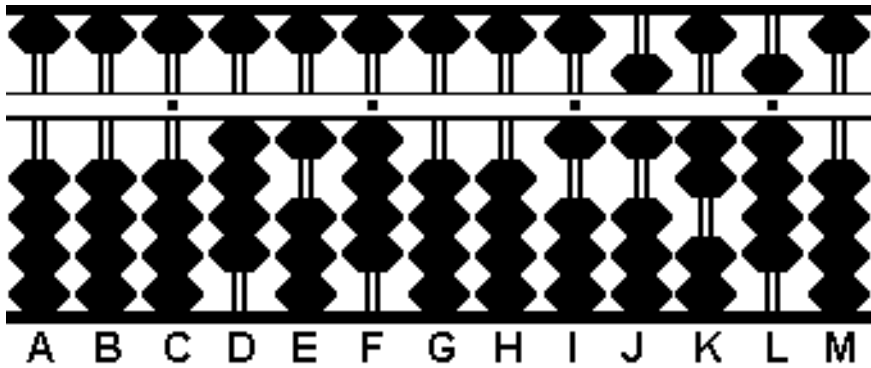
Now the frame looks like:
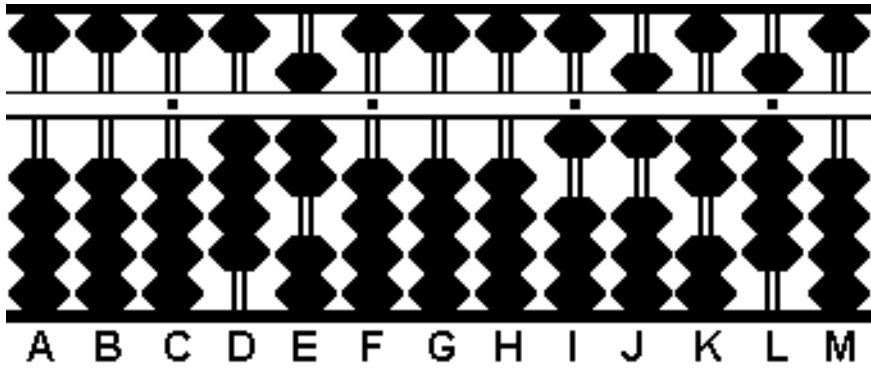

```
Step 4
A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
0  0  0  4  7  0  0  0  6  5  2  9  0
            ( 7 )
                        -  4  9
0  0  0  4  7  0  0  0  1  6  2  9  0
```

Step 5: Next, double the 7 on rod E to equal 14. Place 14 on rods E & F.

This produces the following frame:


```
Step 5
A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
0  0  0  4  7  0  0  0  1  6  2  9  0
         +  7
0  0  0  4  1  4  0  0  1  6  2  9  0
```

Step 6: Now divide 414 on rods D, E & F into 1629, on the right. It looks like 414 goes into 1629 three times with a remainder. Place 3 on rod G and multiply it by the 4 on rod D to equal 12. Subtract 12 from 16 on rods I & J. leaving 4.

Step 6(a)**:** Now multiply 3 by the 1 on rod E to equal 3 and subtract 3 from rod J.

Step 6(b)**:** Next multiply 3 by the 4 on rod F to equal 12 and subtract 12 from rods J & K.

The result after steps 6, 6(a) and 6(b)
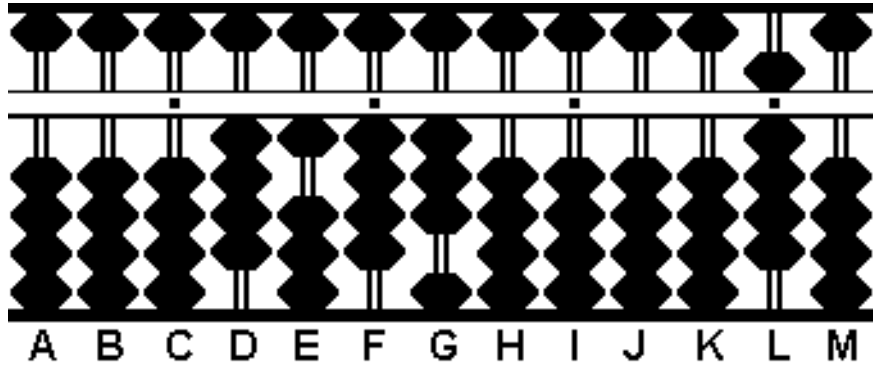

```
Steps 6, 6(a), 6(b)
A B C D E F G H I J K L M
        .       .       .       .
0 0 0 4 1 4 0 0 1 6 2 9 0
                3
              – 1 2
0 0 0 4 1 4 3 0 0 4 2 9 0
                  – 3
0 0 0 4 1 4 3 0 0 1 2 9 0
                  –1 2
0 0 0 4 1 4 3 0 0 0 0 9 0
```

Step 7: The rest is easy. All that's left in this step is to mentally square the 3 and subtract the product from the remainder on the right. 3 x 3 = 9. Subtract 9 from rod L.

Now we've got:
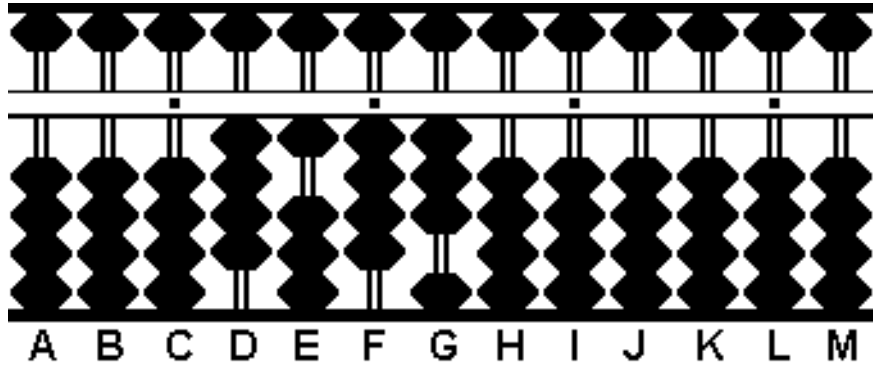

```
Step 7
A B C D E F G H I J K L M
        .       .       .       .
0 0 0 4 1 4 3 0 0 0 0 9 0
                (3)
                      – 9
0 0 0 4 1 4 3 0 0 0 0 0 0
```

Step 8: Nearly done. In steps 3 and 5 we doubled portions of the root number. Now they must be halved to their original numbers. Take half of 4 on rod D to equal 2. Take half of 14 on rods E & F to equal 7 and place it on rod E. Move the 3 on G to rod F. And there's the answer.

The final answer **273** on rods D, E & F.
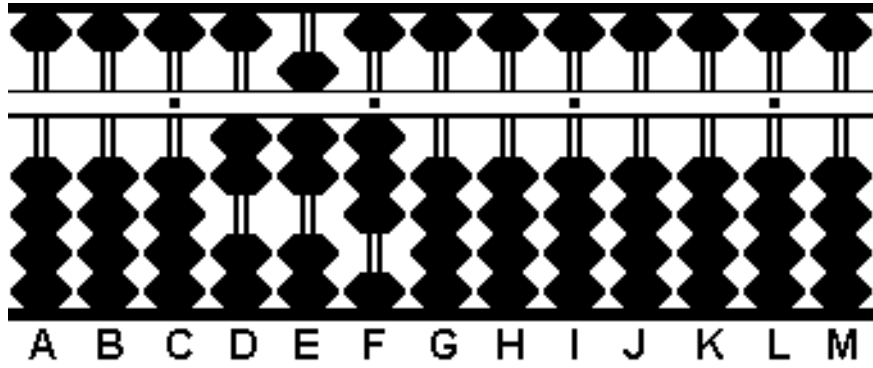
$$\sqrt{74529} = 273$$

Example 3. √ 39.3129 = ?

Step 1: This example involves a decimal. Set 39.3129 on the right hand side of your abacus. This is the square number. If there's room, pair it off 39. 31 29 or do it mentally. The first group is the whole number 39 on rods H & I. From this we know the answer will have one whole number. Anything that follows will be a decimal.

This yields the resulting frame:
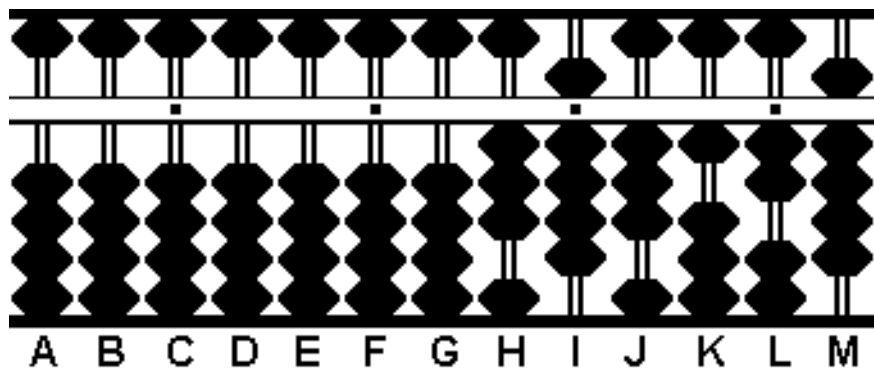

```
Step 1
A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
0  0  0  0  0  0  0  3  9  3  1  2  9
```

Step 2: Now look at the first group of numbers. (In this case the 39 on rods H & I.) Find the largest perfect square that is less than or equal to 39. The answer is 36. Subtract 36 from 39 leaving 3. Set the square root of 36 (namely 6) somewhere on the left-hand side of the abacus, in this case on rod C. This forms the first part of the root number.

Now the frame looks like:
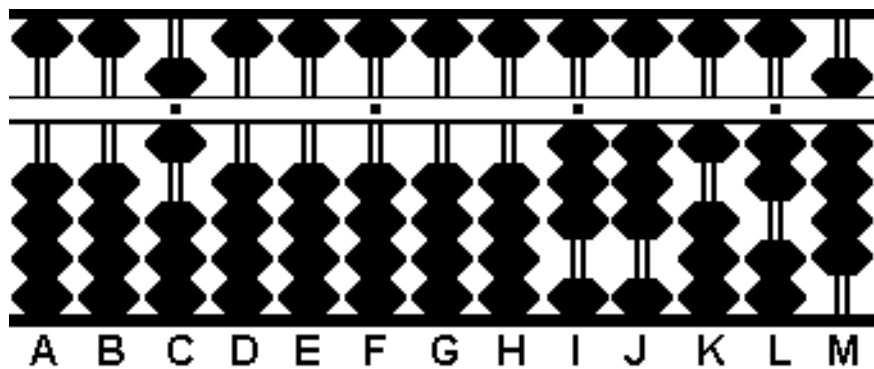

```
Step 2
A  B  C  D  E  F  G  H  I  J  K  L  M
         .        .        .        .
0  0  6  0  0  0  0  3  9  3  1  2  9
                     -  3  6
0  0  6  0  0  0  0  0  3  3  1  2  9
```

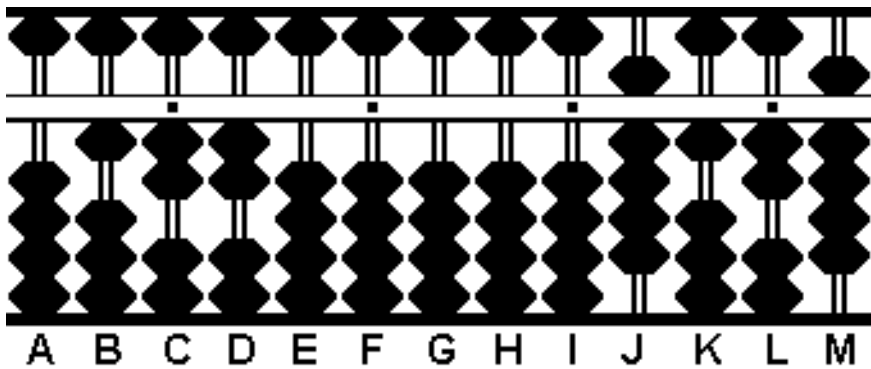**Step 3:** Double the 6 on rod C to equal 12 and place 12 on rods B & C.

Now there's:
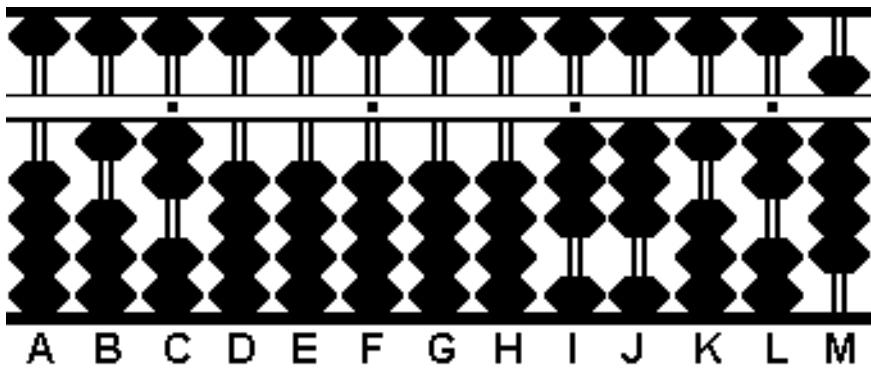

```
Step 3
A B C D E F G H I J K L M
        .       .       .       .
0 0 6 0 0 0 0 0 3 3 1 2 9
  + 6
0 1 2 0 0 0 0 0 3 3 1 2 9
```

Step 4: Divide 12 into 33 on rods I & J. 12 goes into 33 twice with a remainder. Place 2 on rod D. Multiply 12 x 2 to equal 24 and subtract 24 from 33. That leaves 9 on rod J.
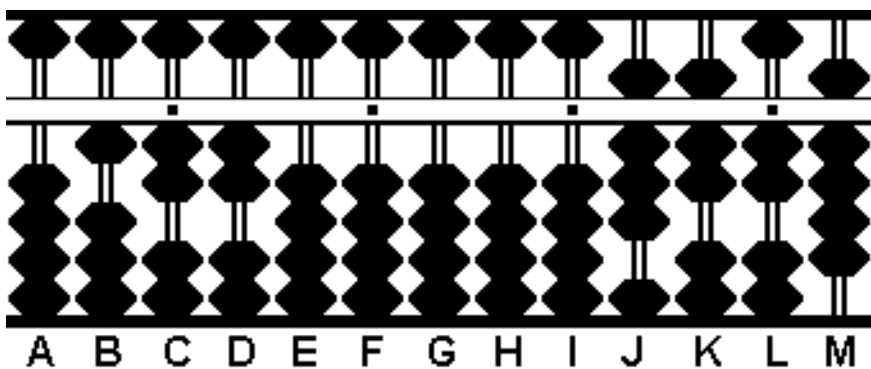
This produces the following:


```
Step 4
A B C D E F G H I J K L M
        .       .       .       .
0 1 2 0 0 0 0 0 3 3 1 2 9
      2
                  - 2 4
0 1 2 2 0 0 0 0 0 9 1 2 9
```

Step 5: Mentally square the 2 on rod D and subtract the product from the remainder on the right. 2 x 2 = 4. Subtract 4 from rod K.

The result:


```
Step 5
A B C D E F G H I J K L M
        .       .       .       .
0 1 2 2 0 0 0 0 0 9 1 2 9
      (2)
                    - 4
0 1 2 2 0 0 0 0 0 8 7 2 9
```

Step 6: Double the 2 on rod D to equal 4 leaving 124 on rods B, C & D.

Now the frame looks like:
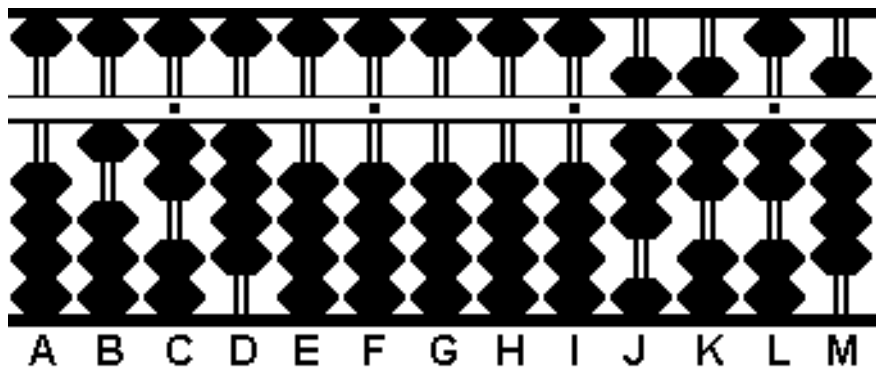

```
Step 6
A  B  C  D  E  F  G  H  I  J  K  L  M
      .        .        .        .
0  1  2  2  0  0  0  0  0  9  1  2  9
      + 2
0  1  2  4  0  0  0  0  0  8  7  2  9
```

Step 7: Now divide 124 on rods B, C & D into 8729 on the right. It looks like 124 goes into 8729 seven times with a remainder. Place 7 on rod E and multiply it by the 1 on rod B to equal 7. Subtract 7 from 8 on rod J leaving 1.

Step 7(a): Multiply 7 by 2 on rod C to equal 14. Subtract 14 from rods J & K.

Step 7(b): Multiply 7 by 4 on rod D to equal 28. Subtract 28 from rods K & L.

Now we've got:
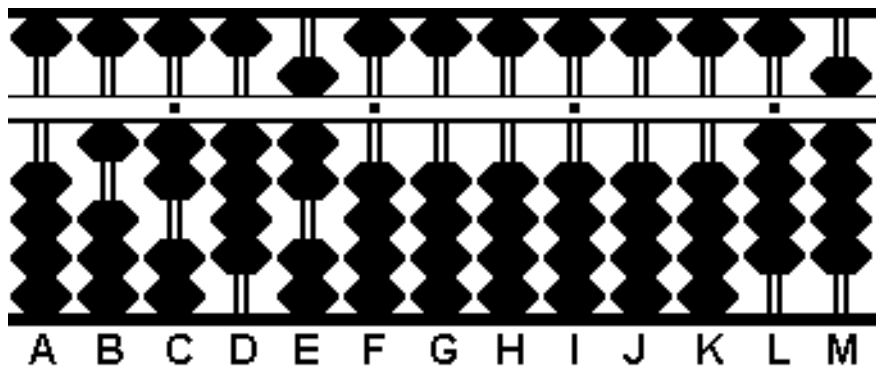

```
Steps 7, 7(a), 7(b)
A  B  C  D  E  F  G  H  I  J  K  L  M
      .        .        .        .
0  1  2  4  0  0  0  0  0  8  7  2  9
            7
                              - 7
0  1  2  4  7  0  0  0  0  1  7  2  9
                            - 1  4
0  1  2  4  7  0  0  0  0  0  3  2  9
                              - 2  8
0  1  2  4  7  0  0  0  0  0  0  4  9
```

Step 8: Almost there. Mentally square the 7 on rod E and subtract the product from the remainder on the right. 7 x 7 = 49. Subtract 49 from rods L & M.

With the following results:
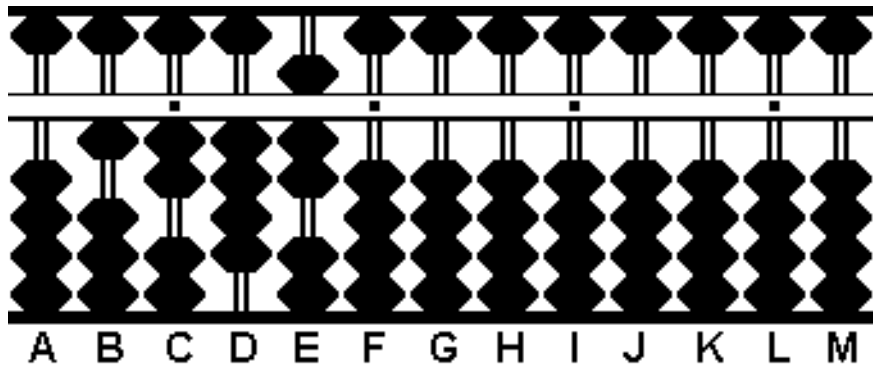
```
Step 8
A B C D E F G H I J K L M
        .     .     .     .
0 1 2 4 7 0 0 0 0 0 0 4 9
          (7)
                    - 4 9
0 1 2 4 7 0 0 0 0 0 0 0 0
```
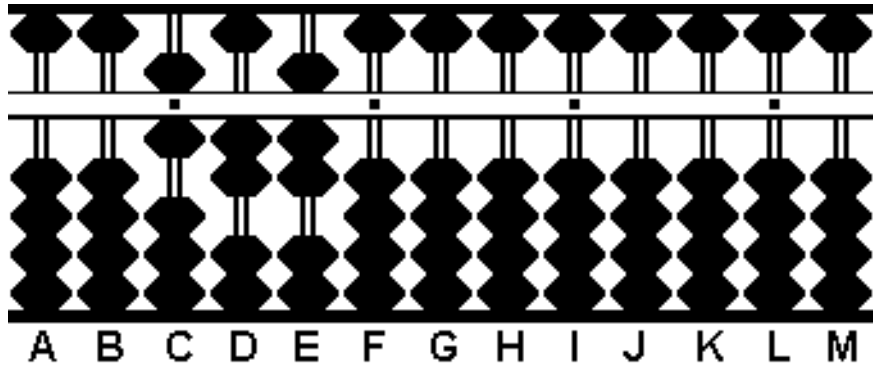
Step 9: The last step. In steps 3 and 6 we doubled portions of the root number. Now they must be halved to their original numbers. Take half of 12 on rods B & C to equal 6. Place 6 on rod C. Take half of 4 on rod D to equal 2. And there's the answer.

The answer **6.27** on rods C, D & E.

$$\sqrt{39.3129} = 6.27$$

Totton Heffelfinger
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⬡ WORKING WITH DECIMALS (contributed by Edvaldo Siqueira)

The following is a method used by Professor Fukutaro Kato, a Japanese soroban teacher living in Brazil in the 1960's. The method was published in the Professor's book, *SOROBAN pelo Método Moderno* which, when translated from Portuguese means *SOROBAN by the Modern Method*

This is an explanation on how to apply "Professor Kato's Method" for placing correctly the *multiplicand* or the *dividend* on the soroban, in order that the unit digit of the *product* or the *quotient* will fall always on a previously chosen unit column. Bear in mind that both arithmetic operations should be carried out according Kojima's standard methods for division & multiplication.

For the sake of clarity, let's split the following in 4 short sections: A) HOW TO COUNT DIGITS; B) NUMBERS ON THE SOROBAN; C) MULTIPLICATION   D) DIVISION

A) COUNTING DIGITS

i) If the 2 pairs of numbers *multiplicand & multiplier* or *dividend & divisor* are mixed numbers (those with a whole part and decimal places), consider in each one only the digits in their whole part. So disregard any decimal digit, after the point, if any.

Ex:

```
3.105......has 1 digit;
47.02......has 2 digits
308.41.....has 3 digits
1000.......has 4 digits and so on
```

ii) When the number is a pure decimal, consider (count) only those trailing zeros (if any) before the first significant digit attributing to the total a negative sign. Disregard the zero before the point

Ex:

```
0.00084.....has -3 digits
0.40087....has  0 digits
```

```
0.01010.....has -1 digit
```

B) NUMBERS ON THE SOROBAN

```
<----A----B----C----D----E---*F*---G----H----I----J-->
<...+6...+5...+4...+3...+2...+1....0...-1...-2...-3..>
```

Let's associate an orientating axis extending from right to left of the soroban rods, with the axis positive number 1 corresponding to the *unit rod* (*F*). That correspondence is paramount to the correct application of this method! Obviously, the rod chosen to be carry the unit can be anyone - towards the center of the soroban - provided it is marked with a dot. To the left of *F* (rods E, D, C ...) we have positive values; to its right, Zero and negative values, as shown. Imagine the above diagram extending to both sides as far as necessary to work comfortably according Kojima's. You will soon note that when the absolute value of N "large" (say, > 4), the other printed dots on the bar are handy in choosing the appropriate rod for setting the multiplicand highest order digit.

▬ MULTIPLICATION

a) *RULES TO FOLLOW*

Compute the Number: N = # digits on multiplicand + # digits on multiplier + 1

Note: Before computing N, please note that number 1 should always be added to the # of digits on the multiplier.

b) *EXAMPLES*

```
30 x 8..............N = 2 + 1 + 1 = 4
2 x 3.14............N = 1 + 1 + 1 = 3
12 x 0.75...........N = 2 + 0 + 1 = 3
0.97 x 0.1..........N = 0 + 0 + 1 = 1
0.5 x 8.14.........N = 0 + 1 + 1 = 2
0.03 x 0.001.......N = -1 + (-2) + 1 = -2
```

c) *SOME SAMPLE QUESTIONS*

```
<----A----B----C----D----E---*F*---G----H----I----J-->
<...+6...+5...+4...+3...+2...+1....0...-1...-2...-3..>
```

i) 40.0 x 0.225 = 9,
N = 2 + 0 + 1 = 3   ----> set 4 on D...product 9 will fall on *F*

ii) 75 x 12 = 900
N = 2 + 2 + 1 = 5   ----> set 75 on BC...After the multiplication process is carried out, you'll be left with only a 9 placed on rod D; but as *F* carries always the unit result digit, the product must be automatically read: 900.

iii) 0.400 x 0.025 = 0.010
N = 0 + (-1) +1 = 0   ----> set 4 on G...(From the product = 0.01, you only see of course the number 1 on rod H, which should be read as 0.01 - because 0 (= the unit), is as always located on *F*.

iv) 2.6 x 3.5 = 9.1
N = 1 + 1 + 1 =3   ----> set 26 on DE...(product 91 will fall on FG; read as 9.1).

With practice, one will be able to compute N mentally and soon you should be using this alternative method correctly and confidently. The bonus is at the same time you will learn how to multiply decimal numbers on the soroban.

Quiz: From the "Examples" section above), choose the soroban appropriate rod to place the highest order digit of the multiplicand; do again each exercise after computing mentally its expression for N; eventually do on your soroban each multiplication suggested.

DIVISION

a) *RULES TO FOLLOW*

Compute the Number: N = # digits on dividend MINUS [# digits on divisor + 1]

Note: Before computing N, please note that number 1 should always be added to the # of digits on the divisor and also always remember that the operation between parenthesis must me carried out FIRST. If you need to, refresh your memory revisiting that old math book that taught you how to algebraically add negative numbers, many years ago. Or ask your son for advice. **:)** That is the burden of this method, I admit.

*****Further rules for placing the quotient digit (never FORGET THIS ALSO)*****

ai) If the digit of the dividend is *greater or equal* to the corresponding digit of the divisor, *skip one rod* to the left of the of this class of the dividend to put the corresponding quotient digit;

aii) If the digit of the dividend is *less than* the corresponding digit of the divisor, put the quotient digit *next to* the highest class you are working on the dividend.

b) *EXAMPLES*

```
32 ÷ 8..............N = 2 - (1 + 1) = 0
5 ÷ .25.......... . N = 1 - (0 + 1) = 0
6.28 ÷ 2............N = 1 - (1 + 1) = -1
12 ÷ 0.75..........N = 2 - (0 + 1) = 1
3.2 ÷ 0.0016.......N = 1 - (-2 + 1) = 2
0.03 ÷ 0.001.......N = -1 - (-2 + 1 = 0
```

c) *SOME SAMPLE QUESTIONS*

```
<----A----B----C----D----E---*F*---G----H----I----J-->
<...+6...+5...+4...+3...+2...+1....0...-1...-2...-3..>
```
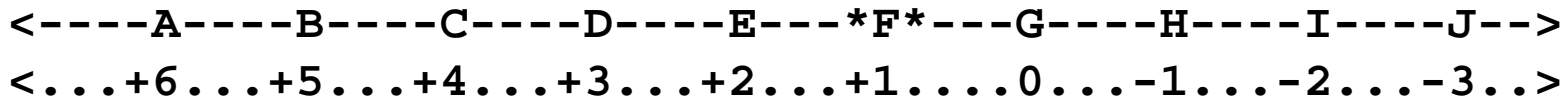
i) 625 ÷ 5 = 125
N = 3 - [1 + 1] = 1   ----> set the dividend (625) on FGH and place the quotient on DEF, (rule ai). The "5" from the quotient will automatically be placed on rod *F* (the unit).

ii) 0,02 ÷ 0.8 = 0.025
N = -1 - [-0 + 1] = -2   ----> set dividend (2) on I and place the quotient digits (25) on HI, (rule aii). As *F* - which always holds the unit (0.) - and G are both empty, one should automatically read 0.025.

iii) 0.08 ÷ 0.002 = 40
N = -1 - [-2 + 1] = -1 - [-1] = -1 +1 = 0   ----> set dividend (8) (disregard the zeros) on G and the calculated quotient 40 on EF (rule ai) - only its digit 4 will be set, but result reads 40.

iv) 86 ÷ 43 = 2
(Mentally) N = -1   ----> set dividend 86 on HI. As first digit of the dividend (8) is greater than first digit of the divisor (4), their quotient will skip one column to the left of this digit of the dividend to place the digit for the quotient. "8 / 4 = 2, which will be set at *F*, (rule ai).

v) 860 ÷ 43 = 20
(Mentally) N = 0   ----> set dividend 860 on GHI. the quotient 2 should be set at E. The soroban operator will interpret this result as 20, because rod F* always carries the unit.

Note: Take some time to compare the quotients digits on the last two examples. One sees for both only number 2. BUT they are placed at DIFFERENT rods. The former is at the very unit rod *F*. The latter is at E and has a unit digit after it, the 0 on rod *F*. So, even without evaluating beforehand their order or magnitude, the student is able to reach the correct answer for each example.

Below find 3 links to pages that have been scanned from Fukutaro Kato's book. These pages will further illustrate how to place correctly the *dividend & quotient digit* on the soroban using the method.

- **Page 1**
- **Page 2**
- **Page 3**

With practice, one will be able to compute N mentally and soon you should be using these alternative methods correctly and confidently. The bonus is at the same time you will learn how to multiply and divide decimal numbers on the soroban.

And a final advice: as you have observed Fukutaro Kato's method allows one to reach the exact result without having to evaluate beforehand the product and quotient order of magnitude, while operating both with integers or decimal numbers! But, if you are already familiar with other system, please stick to it. I am not advocating the one here exposed is better to any other. All I can say that, at least for me, I found it easier to grasp than the other one taught by Kojima, which I never learned :)

Good luck!

Edvaldo Siqueira
Rio de Janeiro, Brazil

# ⬡ NEGATIVE NUMBERS (contributed by Torsten Reincke)

The problem

Negative numbers appear when subtracting a larger number from a smaller one, like in 23-61=-38. These negative numbers can not be displayed on the abacus directly since it is impossible to place e.g. minus 3 beads on a rod. The lack of a minus sign is circumvented by displaying negative numbers indirectly using the concept of complementary numbers. These are sometimes also called supplementary numbers.

Complementary numbers

For example, the complementary number of 3 with respect to 10 is 7. For 4 it is 6, for 8 it is 2. If you add two complementary numbers you always get a number that is a power of 10 (that is 10, 100, 1000, 10000 and so on). This means for example that the complementary number of 3 with respect to 100 is 997. So how do we use this on the abacus? If an arbitrary number is currently set on the abacus, you can immediately read the corresponding complementary number by looking at the beads that are not moved toward the beam but towards the upper and lower border of the frame:
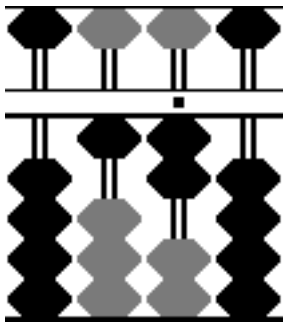
The number 12. The complement is 88 (represented by the grey beads).
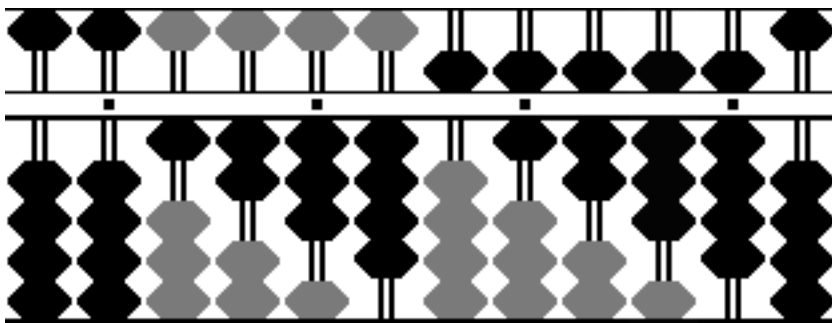
The number 123456789. The complement is 87654321 (again represented by the grey beads).

Note: *You have to add 1 to the last (i.e. rightmost) nonzero rod.*
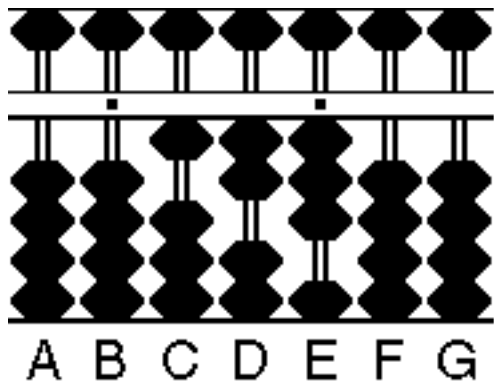
Subtraction using complementary numbers

Let us now solve the original problem, 23-61, using this concept. Place 23 on the abacus. Trying to subtract 61, we immediately see that this is impossible.
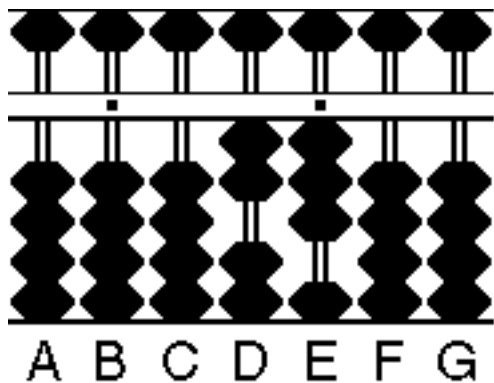
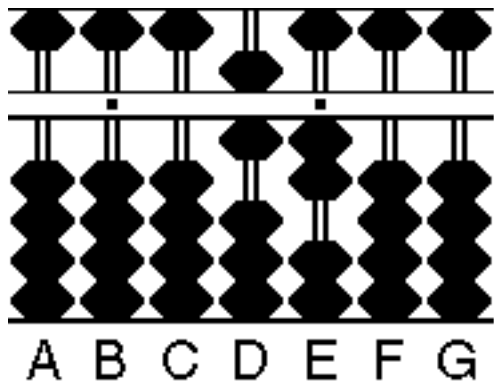The number 23. We can not subtract the 6 (from 61) on rod D.

[ABCDEFG -> 0002300].

Now we use a trick. We borrow 100 by adding 1 on rod C:

We borrowed 100 (i.e. added 1 on rod C).

Now the number is large enough to subtract 61. The result is shown below. The result of the problem is now the complementary number of 62 with respect to the 100 we borrowed a second ago. So the result is -38.

123 - 61 = 62. The result is the complement: -38.

We should save us one additional step in the example above. Instead of first adding 100 and then subtracting 60 (i.e. subtract 6 on rod D) we just add the complement of 6 (with respect to 10) on rod D instead and then subtract the 1 on rod D. Again, instead of adding 100 just add the complement on the rod where you need it. It is just like thinking: "Oh, I need to subtract 6 from 2. That's impossible. So I just add the complement of 6 here (that is 4) and remember that my result will become a negative number so I have to read the complementary number from the abacus, when I'm finished".

Golden rule 1: *When a digit is too large to be subtracted, add its complement instead.*

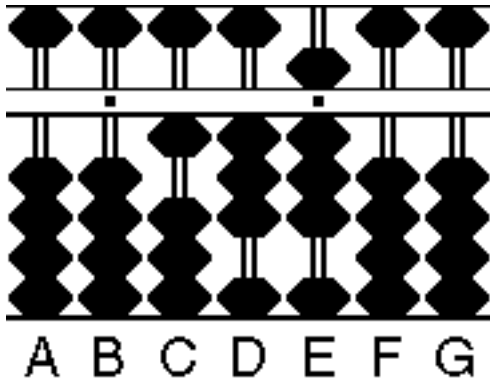**Golden rule 2:** *Using complements makes the calculation negative.*

Next example: 45-97+82
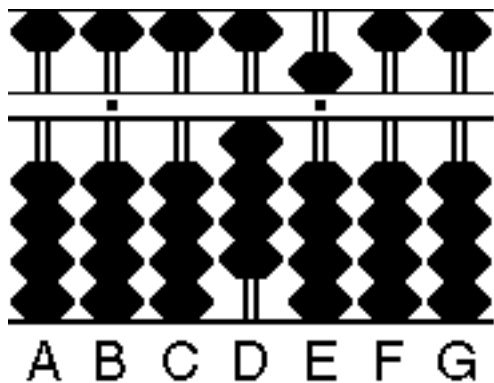
45.
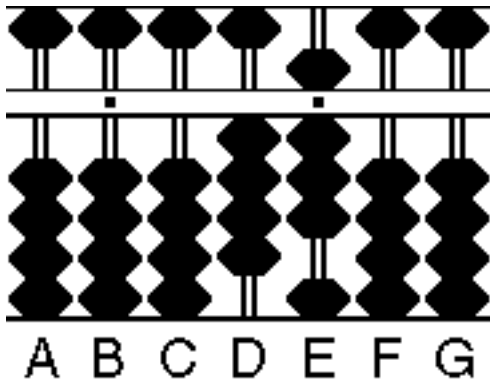
We subtracted 97 (effectively borrowing 1 on rod C) This intermediate result is thus -52.


We added 82.

Here we have a number that is larger than the 100 we borrowed. So we can now safely subtract the 100 again and get a positive result that is 30. This is something we have to be aware of, when the calculation continues with other numbers that make the result positive again. So we have to remember on which rod we borrowed 1 (here rod C) and if we can subtract the 1 from the result we do it and get a positive number again. Note that the complement of 130 (=870) in the figure above would be the complement with respect to 1000 and we never borrowed 1000 but 100 and the result (870) is definitely wrong. (But of course, the complement of 130 with respect to our borrowed 100 is -30, and that is the correct result.)

Let's take another example: 47-83-3389
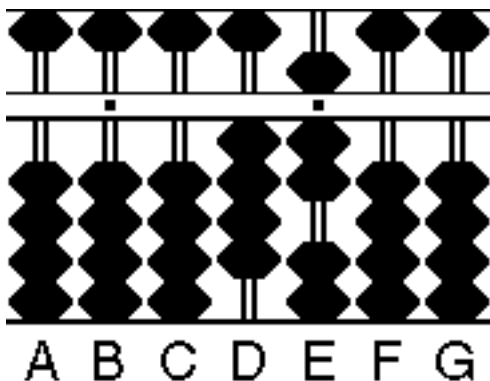
47.
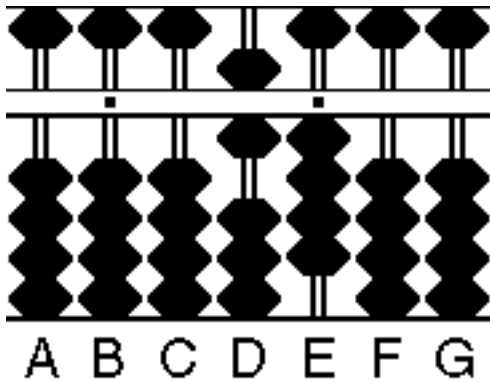
We subtracted 83 (effectively borrowing 1 on rod C).

We have now already borrowed 100 but for the next step this is not enough. Ok, so we have to borrow again, but how much? Note here, that the complement on the abacus is always the complement to a power of 10 (10, 100, 1000, and so on). So if we now just borrowed another 10000 we would get a result which is the complement of 100+10000=10100. We cannot read this complement from the abacus. But if we borrow 9900 instead, the total amount borrowed will be 10000 and we are fine.
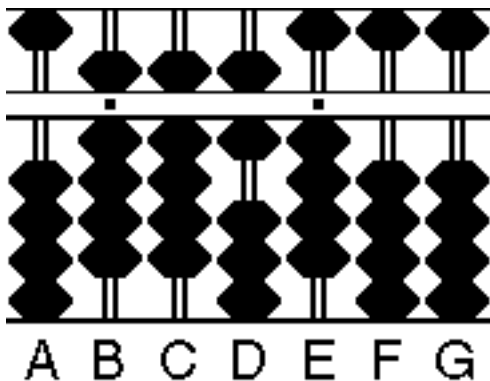
We borrowed another 9900.
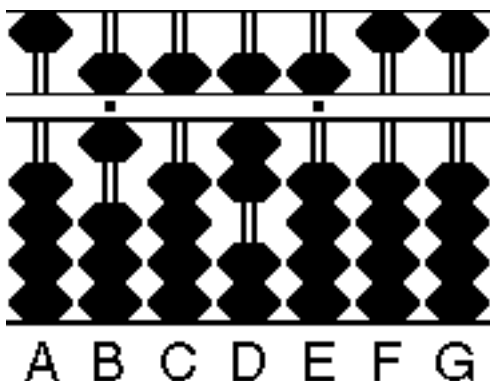
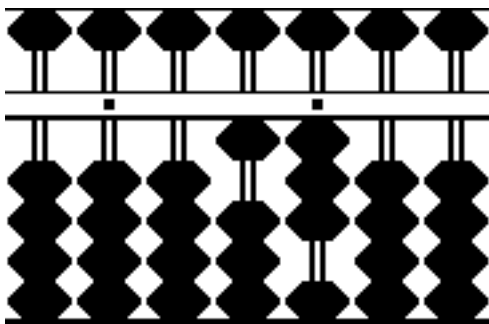We subtracted 3389. The result is the complement: -3425.

The process of borrowing additional amounts in the middle of a calculation is not difficult. You do not have to think 100+9900=10000. Just add 9es on the rods to the left of the current result until there is enough to subtract from and the significant 1 of the borrowed number will be one rod further left.

Golden rule 3: *Add 9es to the left when you need to borrow additional amounts.*
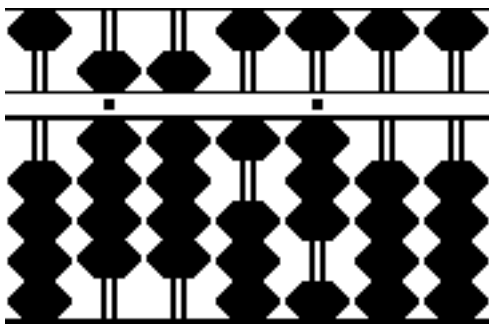
Next example: 12-99-1001
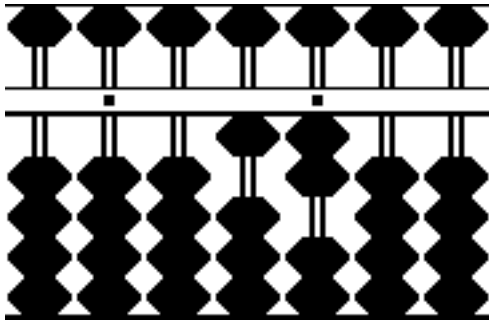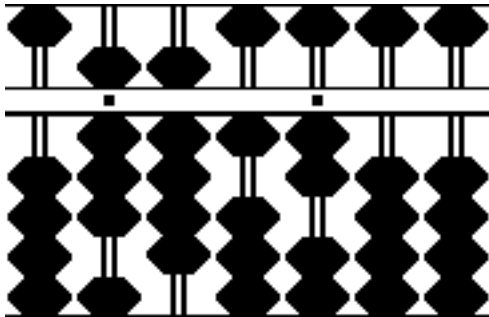
12.

We subtracted 99.

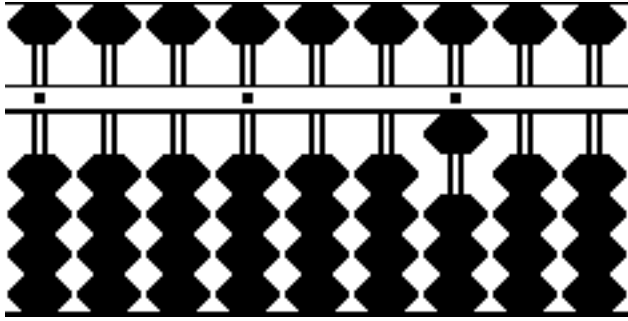This looks like another shortcut. Instead of adding 100-99, just add 1.

We added additional 9es on the left until there is enough to subtract 1001.
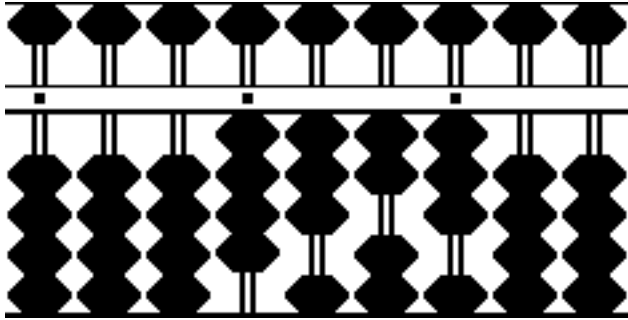
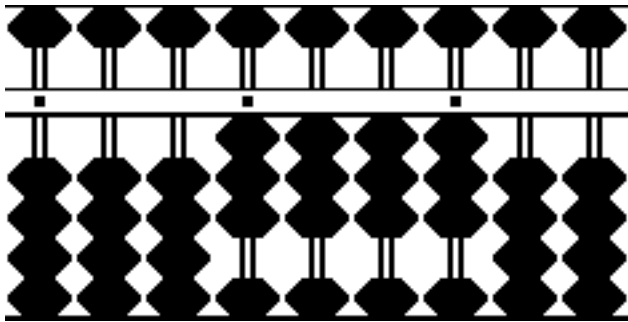We subtracted 1001 to get the total result -1088 as the complement.

Now a longer example: 1-5678-990+6666-54

1.

Subtracted 5678 (borrowing 10000).

Subtracted 990.
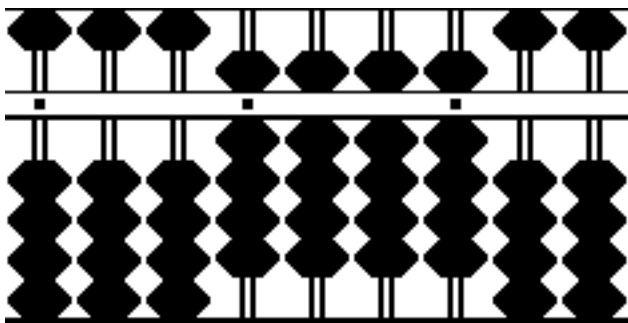
Added 6666.
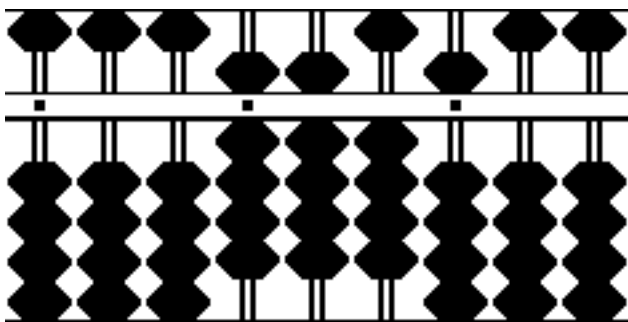
Subtracted 54. The result is -55.

The concept of complementary numbers demonstrated here may sound familiar in knowing bead arithmetics. And in fact you will already have used it in normal addition and subtraction. Consider this: 17+18. First you add 1 on the ten's rod. Then, seeing that you can not add 8 to 7 on the one's rod, you add 1 on the ten's rod and subtract 3 on the one's rod. This is the equivalence of our rule number 1: if you can not add a digit, subtract the complement instead and add 1 on the next rod to the left.

Addition and subtraction also use another complement - the complement of 5. Try the example 2+4 yourself and see how the concept of complementary numbers is used here.

A Tomoe employee called these complements: "friends of 10" and "friends of 5". A nice term for an important concept.

Torsten Reincke
Kiel, Germany

ADVANCED TECHNIQUES FOR
SOROBAN & SUAN PAN
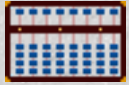
SOROBAN:

Fukutaro Kato Techniques

All of Professor Kato's techniques were taught to me by Edvaldo Siqueira of Rio de Janeiro, Brazil. More than anyone Edvaldo has inspired me to continue studying soroban and to investigate more closely the advanced techniques it has to offer.

Professor Kato's method for Pre-determining the Decimal Place in problems of Division & Multiplication

- Division
- Multiplication
- Combined (Both Division and Multiplication combined in one document)

Professor Kato's method for extracting Square Roots

- Square Roots

Takashi Kojima Techniques

The Elimination of the Initial Digit of a Multiplier Beginning with One

- Elimination

Multiplication by Complementary Numbers

- Complementary

Takashi Kojima's method for extracting Square Roots

- Square Roots
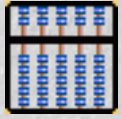
SUAN PAN:

The following techniques were taught to me by two members of the Soroban/Abacus newsgroup; the very knowledgeable Francis Wong who has been instrumental in leading many of us to an understanding of some of the traditional Suan Pan techniques & Torsten Reincke whose contributions bring us all to a greater understanding and more thoughtful use of abacus.

Traditional Chinese Bead Techniques

- Multiplication
- Short Division
- Long Division
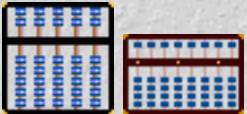
RELATED MATTERS:

Various Help pages:
- **Multiplication and Decimals (**Jeff)
- **Division Revision** (Sarada)
- **Placing Quotient numbers** (New)

The best of the best:
- **Yahoo Soroban/Abacus Newsgroup**
- **Torsten Reincke's Abacist's Guide to the Internet**
- **Collection of Dr. Jörn Lütjens**

And last but by no means least the man who got all of this started for me and for so many others:
- **Dave Bernazzani's Soroban/Abacus Handbook**

Totton Heffelfinger
Toronto Ontario
Canada

🔶 DECIMALS & DIVISION   (contributed by Edvaldo Siqueira)

The following is a method used by Professor Fukutaro Kato, a Japanese soroban teacher living in Brazil in the 1960's. The method was published in the Professor's book, *SOROBAN pelo Método Moderno* which, when translated from Portuguese means *SOROBAN by the Modern Method*

This is an explanation on how to apply "Professor Kato's Method" for placing correctly the *dividend & quotient* on the soroban, in order that the unit digit of the *quotient* always falls on a previously chosen unit rod. In other words, you always know where your unit rod is. Take the example 0.00008 ÷ 0.002 = 0.04 Because you've already chosen your unit rod, what looks like a rather complicated division problem involving lots of zeros becomes a much more manageable 8 ÷ 2 =4

Bear in mind that the arithmetic operation mechanics should be carried out according Kojima's standard method for division.

For the sake of clarity, let's split the following in 4 short sections: A) HOW TO COUNT DIGITS; B) RULE TO FOLLOW; C) NUMBERS ON THE SOROBAN; and D) A FEW EXAMPLES.

A) HOW TO COUNT DIGITS

i) If the *dividend or the divisor* are integers or numbers with a whole part followed by decimal places, take into account in each one only those digits in their whole part. So disregard any decimal digit, after the point, if any.

Ex:

3.105.......has 1 digit;
47.02.......has 2 digits;
308.41.....has 3 digits,
1000........has 4 digits, and so on

ii) When the number is a pure decimal, consider (count) only those trailing zeros (if any) before the first significant digit attributing to the total a negative sign. Disregard the zero before the point

Ex:

0.00084.....has -3 digits
0.40087.....has 0 digits
0.01010.....has -1 digit

B) RULES TO FOLLOW

B1) Compute the Number: N = # digits on dividend MINUS [# digits on divisor + 1]

Note: Before computing N, please note that number 1 should always be added to the # of digits on the divisor and also always remember that the operation between parenthesis must me carried out FIRST. If you need to, refresh your memory revisiting that old math book that taught you how to algebraically add negative numbers, many years ago. Or ask your son for advice :). (See D, 3rd example). That is the burden of this method, I admit.

B2) Further rules for placing the quotient digit (never FORGET THIS ALSO)

i) If the digit of the dividend is *greater or equal* to the corresponding digit of the divisor, *skip one rod* to the left of the of this class of the dividend to put the corresponding quotient digit;

ii) If the digit of the dividend is *less than* the corresponding digit of the divisor, put the quotient digit *next to* the highest class you are working on the dividend.

C) NUMBERS ON THE SOROBAN

```
<----A----B----C----D----E---*F*---G----H----I----J-->
<...+6...+5...+4...+3...+2...+1....0...-1...-2...-3..>
```

Let's associate an orientating axis extending from right to left of the soroban rods with the axis positive number 1 corresponding to the *unit rod* (*F*). That correspondence is paramount to the correct application of this method! Obviously, the rod chosen to hold the unit can be anyone - towards the center of the soroban - provided it is marked with a dot. To the left of *F* (rods E, D, C ...) we have positive values; to its right, Zero and negative values, as shown. Imagine the above diagram extending to both sides as far as necessary to work comfortably according Kojima's. You will soon note that when the absolute value of N "large" (say, > 4), the other printed dots on the bar are handy in choosing the appropriate rod for setting the multiplicand highest order digit.

D) EXAMPLES

i) 625 ÷ 5 = 125
N = 3 - [1 + 1] = 1   ----> set the dividend (625) on FGH and place the quotient on DEF, (rule B2i). The "5" from the quotient will automatically be placed on rod *F* (the unit).

ii) 0,02 ÷ 0.8 = 0.025
N = -1 - [-0 + 1] = -2   ----> set dividend (2) on I and place the quotient digits (25) on HI (Rule B2ii). As *F* - which always holds the unit (0.) - and G are both empty, one should automatically read 0.025.

iii) $0.08 \div 0.002 = 40$

N = -1 - [-2 + 1] = -1 - [-1] = -1 +1 = 0   ----> set dividend (8) (disregard the zeros) on G and the calculated quotient 40 on EF (rule B2i) - only its digit 4 will be set, but result reads 40.

iv) $86 \div 43 = 2$

(Mentally) N = -1   ----> set dividend 86 on HI. As first digit of the dividend (8) is greater than first digit of the divisor (4), their quotient will skip one column to the left of this digit of the dividend to place the digit for the quotient. "8 / 4 = 2, which will be set at *F*, (rule B2i).

v) $860 \div 43 = 20$

(Mentally) N = 0   ----> set dividend 860 on GHI. Quotient (after carrying out the whole division process) will be 2 on E (again rule B2i), AND a 0 (zero) on *F*, because this column always holds the unit digit. Final answer will be: 20

Note: Take some time to compare the quotients digits on the previous two examples. One sees for both only number 2. BUT they are placed at DIFFERENT rods. The former is at the very column *F*. The latter is at E. But this one has a unit digit after it, 0; in fact the former is 20 and the latter is 2 only. So, even without evaluating beforehand their order or magnitude, the student is able to reach the correct answer for each example.

Below find 3 links to pages that have been scanned from Fukutaro Kato's book. These pages will further illustrate how to place correctly the *dividend & quotient digit* on the soroban using the method.

- **Page 1**
- **Page 2**
- **Page 3**

With practice, one will be able to compute N mentally and soon you should be using this alternative method correctly and confidently. The bonus is at the same time you will learn how to divide decimal numbers on the soroban.

And a final advice: as you have observed Fukutaro Kato's method allows one to reach the exact result without having to evaluate beforehand the quotient order of magnitude, while operating both with integers or decimal numbers! But, if you are already familiar with other system, please stick to it. I am not advocating the one here exposed is better to any other. All I can say that, at least for me, I found it easier to grasp than the other one taught by Kojima, which I never learned :)

Good luck!

Edvaldo Siqueira
Rio de Janeiro, Brazil

🔶 DECIMALS & MULTIPLICATION   (contributed by Edvaldo Siqueira)

The following is a method used by Professor Fukutaro Kato, a Japanese soroban teacher living in Brazil in the 1960's. The method was published in the Professor's book, *SOROBAN pelo Método Moderno* which, when translated from Portuguese means *SOROBAN by the Modern Method*

This is an explanation on how to apply "Professor Kato's Method" for placing correctly the *multiplicand* on the soroban,in order that the unit digit of the *product* will fall always on a previously chosen unit rod. Bear in mind that the arithmetic operation should be carried out according Kojima's standard method for multiplication.

For the sake of clarity, let's split the following in 4 short sections: A) HOW TO COUNT DIGITS; B) RULE TO FOLLOW; C) NUMBERS ON THE SOROBAN; and D) A FEW EXAMPLES.

A) COUNTING DIGITS

i) If the *multiplicand or the multiplier* are integers or mixed numbers (those with a whole part and decimal places), consider in each one only the digits in their whole part. So disregard any decimal digit, after the point, if any.

Ex:

```
3.105......has 1 digit;
47.02......has 2 digits
308.41.....has 3 digits
1000.......has 4 digits and so on
```

ii) When the number is a pure decimal, consider (count) only those trailing zeros (if any) before the first significant digit attributing to the total a negative sign. Disregard the zero before the point

Ex:

```
0.00084.....has -3 digits
0.40087.....has  0 digits
0.01010.....has -1 digit
```

B) RULES TO FOLLOW

B1) Compute the Number: N = # digits on multiplicand + # digits on multiplier + 1

Note: Before computing N, please note that number 1 should always be added to the # of digits on the multiplier.

B2) EXAMPLES:

```
30 x 8..............N = 2 + 1 + 1 = 4
2 x 3.14...........N = 1 + 1 + 1 = 3
12 x 0.75..........N = 2 + 0 + 1 = 3
0.97 x 0.1.........N = 0 + 0 + 1 = 1
0.5 x 8.14.........N = 0 + 1 + 1 = 2
0.03 x 0.001.......N = -1 + (-2) + 1 = -2
```

C) NUMBERS ON THE SOROBAN

```
<----A----B----C----D----E---*F*---G----H----I----J-->
<...+6...+5...+4...+3...+2...+1....0...-1...-2...-3..>
```

Let's associate an orientating axis extending from right to left of the soroban rods, with the axis positive number 1 corresponding to the *unit rod* (*F*). That correspondence is paramount to the correct application of this method! Obviously, the rod chosen to be carry the unit can be anyone - towards the center of the soroban - provided it is marked with a dot. To the left of *F* (rods E, D, C ...) we have positive values; to its right, Zero and negative values, as shown. Imagine the above diagram extending to both sides as far as necessary to work comfortably according Kojima's. You will soon note that when the absolute value of N "large" (say, > 4), the other printed dots on the bar are handy in choosing the appropriate rod for setting the multiplicand highest order digit.

D) EXAMPLES

i) 40.0 x 0.225 = 9,
N = 2 + 0 + 1 = 3   ----> set 4 on D...product 9 will fall on *F*

ii) 75 x 12 = 900
N = 2 + 2 + 1 = 5   ----> set 75 on BC... After the multiplication process is carried out, you'll be left with only a 9 placed on rod D; but as *F* carries always the unit result digit, the product must be automatically read: 900.

iii) 0.400 x 0.025 = 0.010

N = 0 + (-1) +1 = 0   ----> set 4 on G...(From the product = 0.01, you only see of course the number 1 on rod H, which should be read as 0.01 - because 0 (= the unit), is as always located on *F*.

iv) 2.6 x 3.5 = 9.1
N = 1 + 1 + 1 =3   ----> set 26 on DE...(product 91 will fall on FG; read as 9.1).

With practice, one will be able to compute N mentally and soon you should be using this alternative method correctly and confidently. The bonus is at the same time you will learn how to multiply decimal numbers on the soroban.

Quiz: Try to do the above Examples on B2, choose the soroban appropriate rod to place the highest order digit of the multiplicand; do again each exercise after computing mentally its expression for N; eventually do on your soroban each multiplication suggested.

Good luck!

Edvaldo Siqueira
Rio de Janeiro, Brazil

Kato Fukutaro's Square Roots (contributed by Edvaldo Siqueira)

The following is a method used by Professor Fukutaro Kato, a Japanese soroban teacher living in Brazil in the 1960's. The method was published in the Professor's book, *SOROBAN pelo Método Moderno* which, when translated from Portuguese means *SOROBAN by the Modern Method.*

I will try to expose Kato's ideas for extracting the *square root*. If I remember well, Kojima mentions *en passant* this method. Before the method itself, it is useful to grasp the three following concepts.

I) ONE HALF OF A SQUARED NUMBER (Although one gains speed in learning by heart the table below, its memorization is not strictly necessary while extracting the square root.)

Number	Its Square	Half its Square
1	1	0.5
2	4	2.0
3	9	4.5
4	16	8.0
5	25	12.5
6	36	18.0
7	49	24.5
8	64	32.0
9	81	40.5

II) HOW TO HALF A NUMBER ON THE SOROBAN (WITHOUT DISPLACING THE ORDER)

Here we don't want to displace the order of the numbers on the soroban. Multiplying by 0.5 is the same as dividing by 2 (for some it may be easier to multiply than to divide). As an example lets say we had the number 325, as below on D,E & F. Multiplying 325 x 0.5 = 1625.

A B C D E F G H	A B C D E F G H
0 0 0 3 2 5 0 0	0 0 0 1 6 2 5 0

As an alternative one could think of it this way. Instead of multiplying by 0.5, starting on the *right hand side and working left* take ½ of 5 on F and place the result 2.5 on FG. Take ½ of 2 on E making it 1. Finally take ½ of 3 on D and place the result 1.5 on D & E leaving 1625 on rods DEFG.

III) PLACING NUMBERS ON THE SOROBAN

With regards to setting numbers on the soroban, Professor Fukutaro explains it this way;

a) The radicand has to be set so that the its unit falls always on a
unit rod.

b) When a number is not a perfect square, one must pay attention
that the remainder unit should always fall also on that unit rod.

c) To set the square root (the result), one must follow these
principles:

ci) If the radicand is a complete 2-digit group number,
(like 1225), set its square root highest digit (3) exactly to the
1st. left column adjacent to this radicand

cii) If the radicand is an incomplete 2-digit group number
(like 15129), skip one column to the left to set the square root
highest order digit (1 in this case).

With these notions (I, II & III) in mind, lets try do find the square root of *1225*

Before continuing, it may be helpful to visit these links;
===> *Gary Flom contributed this algebraic explanation of how the method works:* *Gary Flom's Explanation*
===> *Professor Fukutaro's algebraic & graphic explanation :* *Professor Fukutaro's Explanations*
===> *For a more extensive explanation on number placement:* *Number Placement*

Example 1: $\sqrt{1225} = 35$

$$\sqrt[2]{1\,2\,2\,5} \;=\; 3\,5$$

		1	2	2	5	
a		1	2	2	5	
b	3		−9			
c			3	2	5	× 0.5
				2	5	
			1	0		
		1	5			
d	3		1	6	2	5
	5	−1	5			
			1	2	5	
		−1	2	5		
e	3	5		"zero"		

a) Mentally separate the the number 1225 into groups of 2 numbers each, 12 25.

b) Look at the first group (12). Find the largest perfect square that is less than or equal to 12. The answer is 3. (Square this 3 = 9). Subtract 9 from 12. We're left with 325.

c) Take half of 325 (multiply by 0.5). The result is 162.5

d) To find the second digit, divide 3 into 16. The largest integer is 5.

d) Multiply the 1st calculated digit by the 2nd. (3 x 5) and subtract the product 15 from 16. We are left with 12.5.

e) Subtract *half* the square of 5 = 12.5 (see the above table, if you have not already learned it by heart.) We are left with "zero"

That's it! We have just found the square root. It must be 35.

Example 2: $\sqrt{362404}\;=\;602$

$$\sqrt[2]{3\,6\,2\,4\,0\,4} \;=\; 6\;0\;2$$

a) Mentally separate the the number 362404 into groups of 2 numbers each, 36 24 04.

b) Look at the first group (36). Find the largest perfect square that is less than or equal to 36. The answer is 6.. (Square this 6 =36). Subtract 36 from the first group, leaving 02404.

c) Take half of 2404 (multiply by 0.5). The result is 1202

d) To find the second digit, divide 6 into 1. It can't be done so move on to divide 6 into 12. The largest integer is 2.

d) Multiply the 1st calculated digit by the 2nd. (6 x 2) and subtract the product 12 from 12. We are left with 2.

e) Subtract *half* the square of 2 = 2 (see the above table). We are left with "zero"

The answer to the problem is 602.

Example 3: $\sqrt{0.2025} = 0.45$

$$\sqrt[2]{0.2\,0\,2\,5} = 0.45$$

a	0.	2	0	2	5	
b	4	−1	6			
c			4	2	5	× 0.5
				2	5	
			1	0		
			2	0		
d	4		2	1	2	5
		5	−2	0		
			0	1	2	5
			−1	2	5	
e	4	5		"zero"		

a) Mentally separate the the number 0.2025 into groups of 2 numbers each, 0   20 25.

b) Here the first workable group is 20. It's largest perfect square is 4. (Square this 4 =16). Subtract 16 from the first group, leaving 425.

c) Take half of 425 (multiply by 0.5). The result is 212.5

d) To find the second digit, divide 4 into 21. The largest integer is 5.

d) Multiply the 1st calculated digit by the 2nd. (4 x 5) and subtract the product 20 from 21. We are left with 2.

e) Subtract *half* the square of 5 = 12.5 (see the above table). We are left with "zero"

The answer to the problem is 45.

Example 4: $\sqrt{228520} = 478$
*** This is an interesting example because it shows how easy it is to revise an answer in the event of a mistake.***

$$\sqrt[2]{2\ 2\ 8\ 5\ 2\ 0} = 478$$

		2	2	8	5	2	0	
a		2	2	8	5	2	0	
b	4	−1	6					
c			6	8	5	2	0	× 0.5
						0		
					1	0		
				2	5			
			4	0				
			3	0				
d	4		3	4	2	6	0	
		8̸	−3	2				
	7		2	2	6	0		
			+4					
			6	2	6	0		
			−2	4	5			
e	4	7		3	8	1	0	
			8	−3	2			
				−5	6			
					5	0		
					3	2		
f	4	7	8			1	8	× 2
						3	6	

a) Mentally separate the the number 228520 into groups of 2 numbers each, 22 85 20.

b) We know the first number in the answer will be 4. (Square this 4 =16). Subtract 16 from the first group 22, leaving 68520.

c) Take half of 68520 (multiply by 0.5). The result l is 34260

d) To find the second digit, divide 4 into 34. The largest integer is 8. Multiply 4 x 8 and subtract the product 32 from 34. Now we have to subtract the *half* of the square of 8 = 32 (see table). *We don't have 32 to subtract.* The solution? Revise the answer from 8 to 7 and give back 4, leaving 6260. Now we can subtract *half* the square of 7 = 24.5 (see the above table). We are left with 3810

e) To find the third digit, divide 4 into 38. The largest integer is 8. Subtract the product 32 from 38. Next multiply 7 x 8 and subtract its product 56 leaving 50. Subtract *half* the square of 8 = 32 (see the above table), leaving 18.

f) Although this problem could go further, this is as far as the Professor takes us. Instead of continuing, he shows us what to do with a remainder. In step c) we multiplied by 0.5 (divided by 2). Now we need to multiply 18 by 2. (18 x 2 = 36)

The answer to the problem is 478, remainder 36

MORE EXAMPLES:

$$\sqrt[2]{1.0404} = 1.02$$

a		...	1.	0	4	0	4	
b	1		−1					
c			0	0	4	0	4	× 0.5
						2	0	
					0	0		
				2	0	0		
			0	0				
d	1			2	0	2		
	0	2		−2				
				0	0	2		
					0	2		
						−2		
e	1	0	2			"zero"		

$$\sqrt[2]{9.006001} = 3.001$$

a			9.	0	0	6	0	0	1
b	3		−9						
c			0	0	0	6	0	0	1	× 0.5
								0	5
							0	0	
						0	0		
						3	0		
d	3					3	0	0	0	5
		0	0	1		−3			
						0	0		
							0	0	
								0	0	5
								−0	5
e	3	0	0	1			"zero"		

$$\sqrt[2]{\ldots 1\,5\,1\,2\,9} = 1\,2\,3$$

			1	5	1	2	9	
a			1	5	1	2	9	
b	1	-1						
c			0	5	1	2	9	× 0.5
						4	5	
					1	0		
				0	5			
			2	5				
d	1			2	5	6	4	5
		2	-2					
			0	5	6	4	5	
			-2					
e	1	2		3	6	4	5	
		3	-3					
				-6				
				0	4	5		
				-4.	5			
f	1	2	3		"zero"			

$$\sqrt[2]{1\,1.9\,0\,2\,5} = 3.4\,5$$

			1	1.	9	0	2	5	
a		1	1.	9	0	2	5		
b	3		-9						
c			2	9	0	2	5	× 0.5
						2	5	
					1	0		
				0	0			
				5				
		1	0					
d	3		1	4	5	1	2	5
		4	-1	2				
			2	5	1	2	5	
			-8					
e	3	4		1	7	1	2	5
			5	-1	5			
				2	1			
				-2	0			
					1	2	5	
					-1	2	5	
f	3	4	5		"zero"			

$$\sqrt[2]{0.0\,0\,2\,0\,2\,5} = 0.0\,4\,5$$

		0.	0	0	2	0	2	5	
a	0.	0	0	2	0	2	5		
				4	-1	6			
				4	2	5		× 0.5
						2	5	
					1	0		
			2	0				
		4		2	1	2	5	
	5	-2	0					
		0	1	2	5			
		-1	2	5				
b	4	5		"zero"				

PRINTER FRIENDLY EXAMPLES

In the event you need to Print Examples 1, 2, 3 & 4 along with their explanations then you may wish to link to the following;

Edvaldo Siqueira
Rio de Janeiro, Brazil

& Totton Heffelfinger
Toronto Ontario
Canada

Examples from Takashi Kojima's book "Advances Abacus, Theory and Practice"
(All examples are taken from the chapter "Other Methods of Multiplication.")

The Elimination of the Initial Digit of a Multiplier Beginning with One

Multiplication can also be simplified by leaving out the initial digit on the multiplier. The Japanese technical term for this may be translated as *the initial-digit-one-elimination-multiplication.*

Example 1. 75 x 103 = 7,725

Fig. 1

Step 1: Set the multiplicand 75 on DE and the 3 of the multiplier 103 on A. (Fig. 1)

Fig. 2
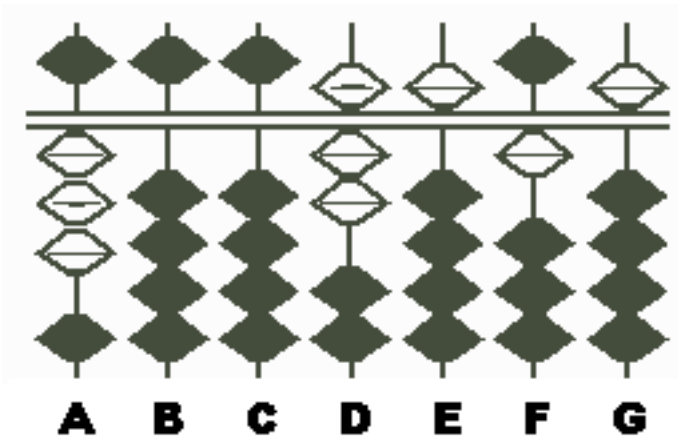
Step 2: Multiply 3 on A by 5 on E, set the product 15 on FG. This makes a total of 7,515 on DEFG. (Fig. 2)

Fig. 3
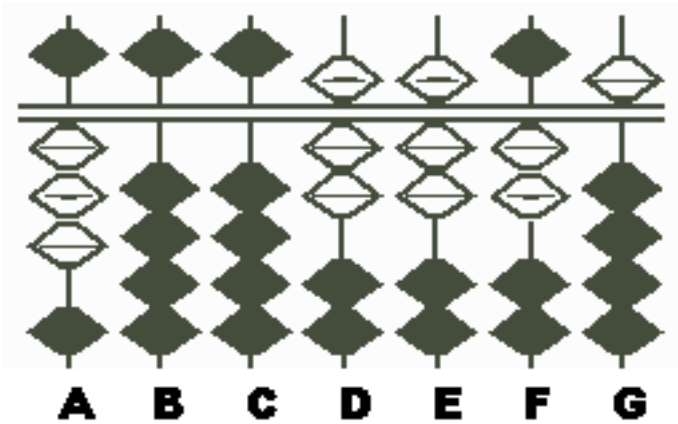
Step 3: Next multiply the 3 on A by the 7 on D, add the product 21 to 51 on EF. This gives you a total of 7725 on EFGH, which is the answer. (Fig.3)

Example 2. 78 x 13 = 1,014

Fig. 1
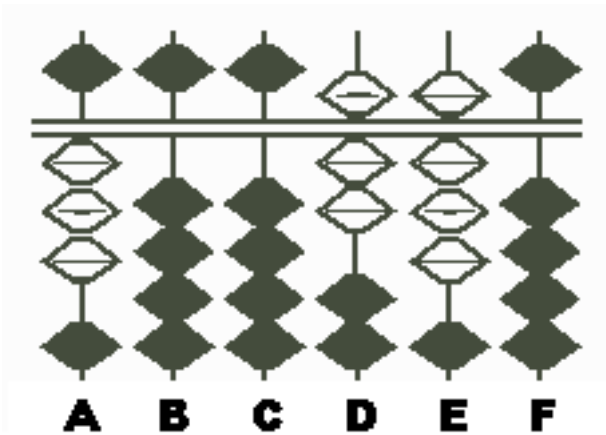
Step 1: Set the multiplicand 78 on DE and the 3 of the multiplier 13 on A. (Fig.1)

Fig. 2
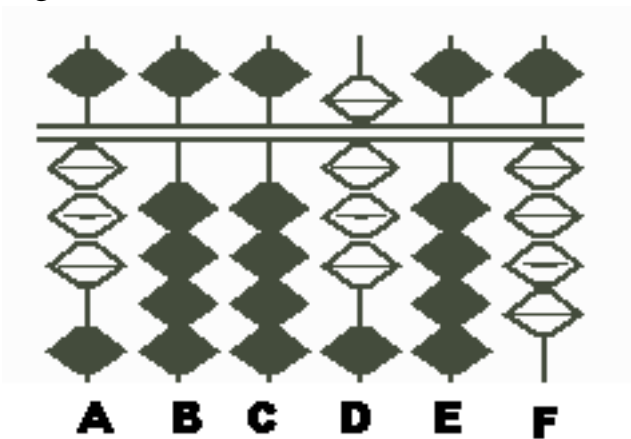
Step 2: Multiplying the 3 on A by the 8 on E, set the product 24 on EF. This makes a total of 804 on DEF. (Fig.2)

(Note: However, you must remember till the next step that the digit which was originally on D was not 8 but 7.)

Fig. 3
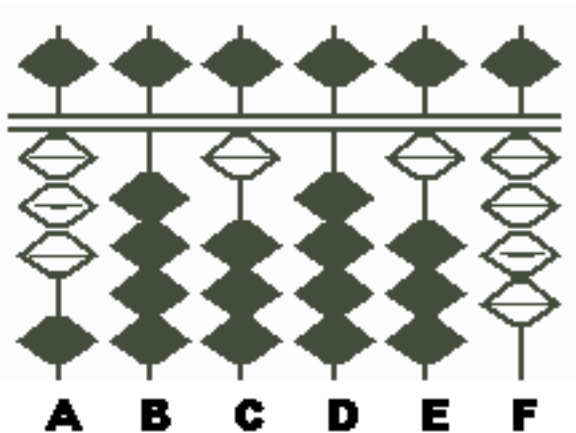
Step 3: Multiplying the 3 on A by the 7, which you remember was on D, set the product 21 on DE. This gives you, on CDEF, a total of 1,014, which is the answer. (Fig.3)

Example 3. An article was bought for $250 and sold at a gain of 6.8%. Find the selling price. Since the selling price is determined by adding the cost and the profit, it can be found by the following multiplication.

$250 X (1 + 0.068) = $250 X 1.068 = $267

There are two methods for working this multiplication. One is the standard method of multiplication and the other the method of eliminating the initial digit one of the multiplier. The latter operation is shown below.

Fig. 1
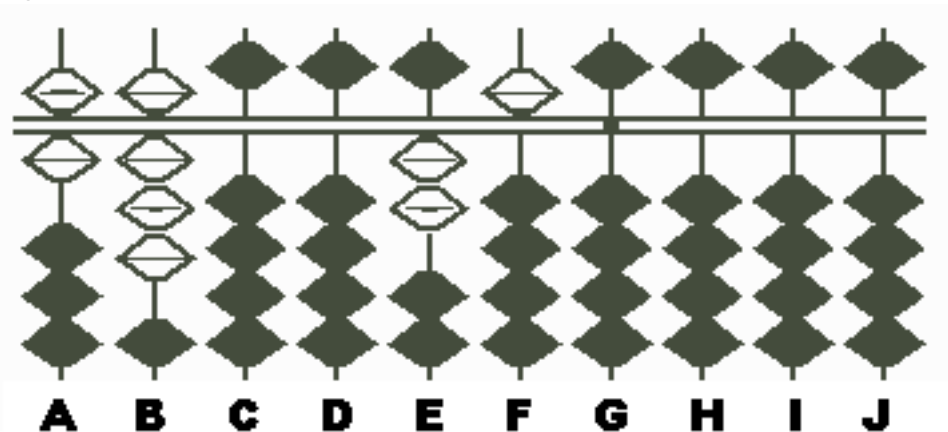
Step 1: Set the multiplicand 250 on EFG, with G as the unit rod, and set the 68 of the multiplier on AB. (Fig.1)

(Note: When the multiplier is a mixed decimal, it is generally advisable to set its unit figure on a unit rod. But experts usually do not bother to, since the digit 1 is not used in calculation. As long as you remember that 68 stands for 1.068, it matters little to set the unit digit of the multiplier on the unit rod.)

Fig. 2
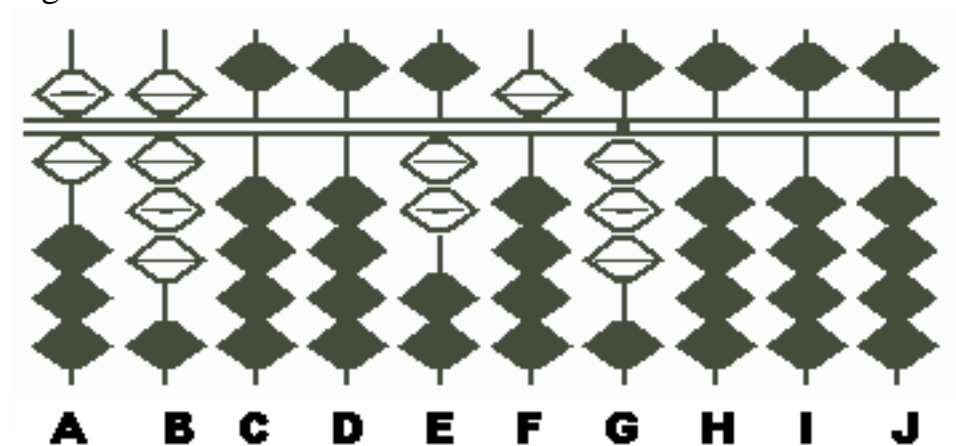
Step 2: Since the multiplier is a four-digit number, suppose that 250 has been multiplied by 1000, producing 250,000 on EFGHIJ, with G as the unit rod. Now multiplying the 6 on A by the 5 on F, set 30 on GH. (Fig.2)

(Note: As both the 6 on A and the 5 on F are in the tens place, the product 30 must be set on GH.)

Fig. 3
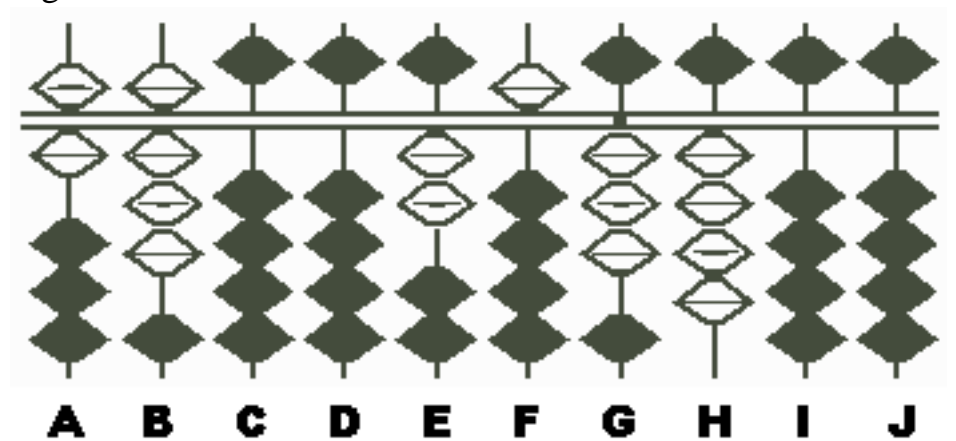
Step 3: Next multiplying the 8 on B by same 5 on F, set the product 40 on HI. This gives you 34 on GH. (Fig.3)

Fig. 4
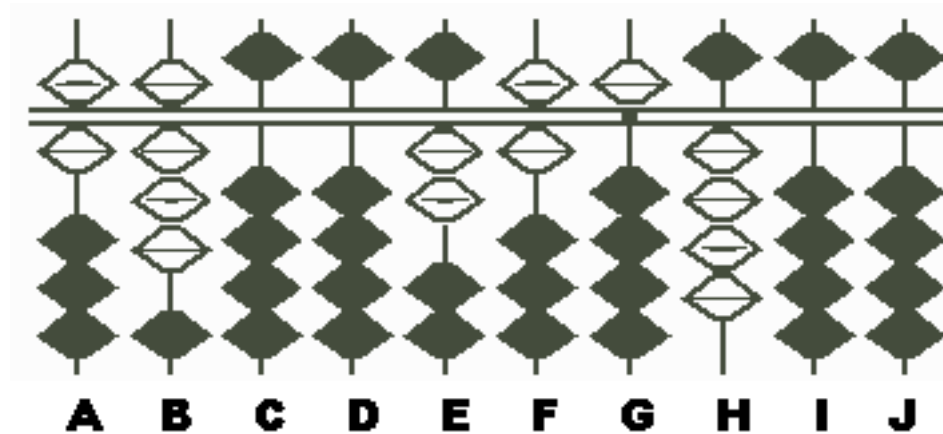
Step 4: Next multiplying the 6 on A by the 2 on E, ass the product 12 to the 53 on FG. This gives you 654 on FGH. (Fig.4)

Fig. 5
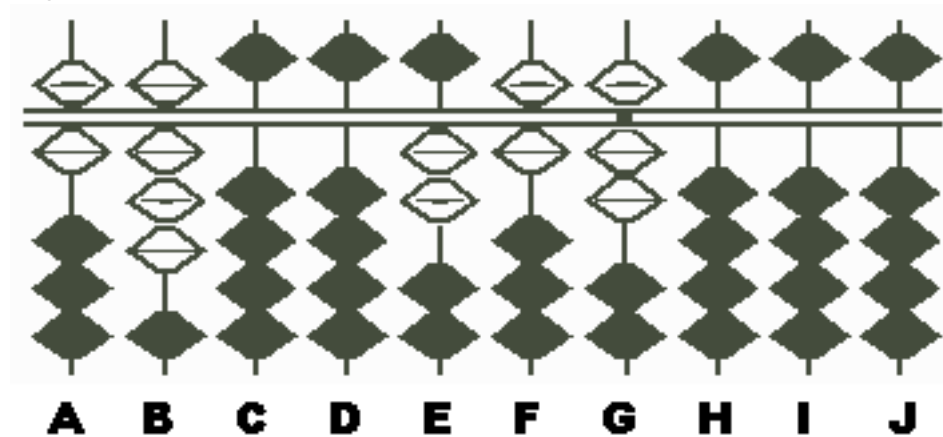
Step 5: Finally multiplying the 8 on B by the same 2 on E, add the product 16 to the 54 on GH. This gives you, on EFG, a total of 267, which is the answer. (Fig.5)

- As written by Takashi Kojima
from his book, "Advances Abacus, Theory and Practice"

Examples from Kojima's book

Examples from Takashi Kojima's book "Advances Abacus, Theory and Practice"

(All examples are taken from the chapter "Other Methods of Multiplication.")

Multiplication by Complementary Numbers

Multiplication can often be simplified by using the complement of a multiplier. For instance, take the problem 26 x 98. The 26 is multiplied by 100 becoming 2,600. From this 26 x 2 is subtracted. In this way the answer 2,548 is obtained. The 2 is, of course, the complement of 98 with respect to 100. This method of computation is better than the ordinary method of multiplication when the the multiplier is a number a little smaller than 100 or 1,000 etc., such as 97 or 996 etc.

Example 1: 26 x 98 = 2,548

Step 1: Set the multiplicand 26 on DE and on A, set 2, the complement of 98 with respect to 100 (Fig 1). When the multiplier is a two-figure number, the multiplicand is regarded as having been multiplied by 100. So in this problem the unit rod of the product shifts to G.

Fig. 1
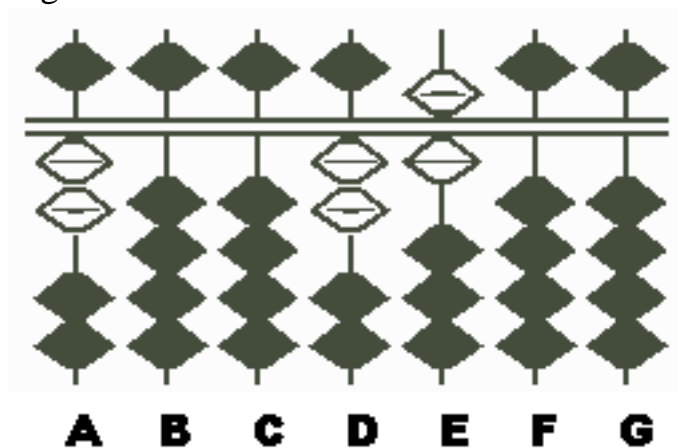
Step 2: Multiplying the 2 on A by the 6 on E, subtract the product 12, which is to be set on FG, from the 6 on E. This leaves 2,588 on DEFG (Fig. 2)
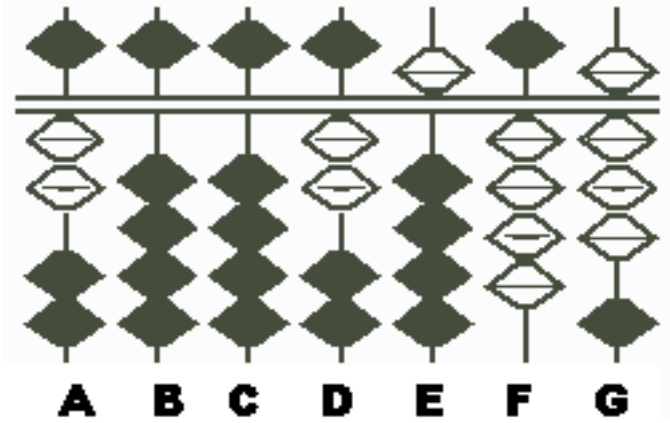
Fig. 2

A B C D E F G
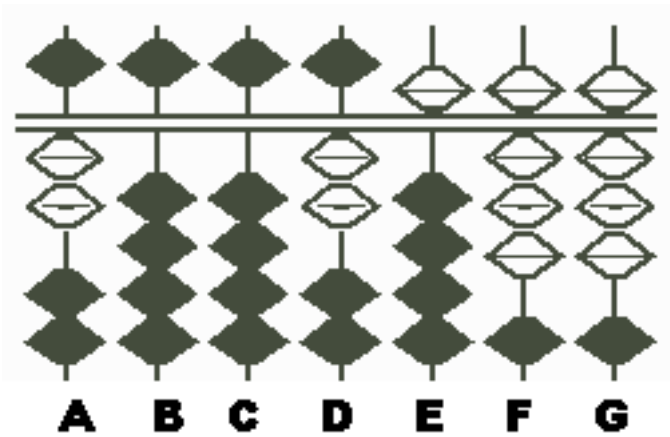
Step 3: Multiplying the 2 on A by the 2 on D, subtract the product 4 from the * on F. This leaves 2,548 on DEFG (Fig.3), the answer.

Fig. 3

A B C D E F G

Example 2: A manufacturer wishes to purchase a piece of machinery priced at $7,250 on which there is a discount of 4% for cash. Find the price he pays in cash. The answer can be found by the following simplified multiplication.

  7,250 x 0.96
=7,250 x (1 - 0.04)
=7,250 - (7,250 x 0.04)
=6,960

Step 1: Set the multiplicand 7,250 on DEFG and on A, set the 4 of 0.04, which is the complement of 0.96 with respect to 1. (Fig.1)

Fig. 1

A B C D E F G H
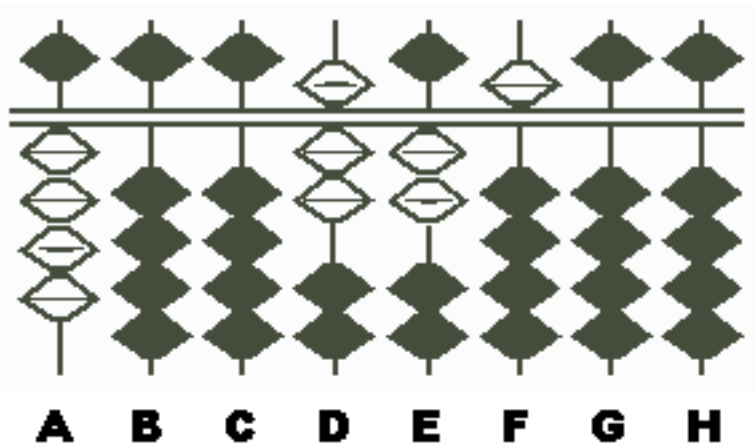
NOTE: On the board you have this problem: 7,250 - (7,250 x 0.04). In this problem, the actual multiplier is 0.96, i.e., a decimal with it's first significant digit in the tenths place. So you may consider that the answer is obtained by keeping the multiplicand as it is and subtracting from it 7,250 x 0.04. You should note that when the multiplier is a decimal with its first significant digit in the tenths place, the unit rod of the product remains that of the multiplicand.

Step 2: Multiplying the 4 on A by the 5 on F, subtract the product 20, which is to be set on GH, from the 5 on F. This leaves 7,248 on DEFG. (Fig.2)
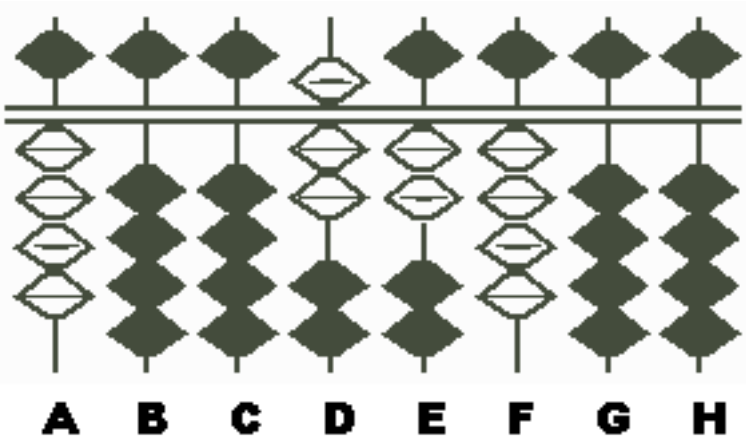
Fig. 2

A B C D E F G H
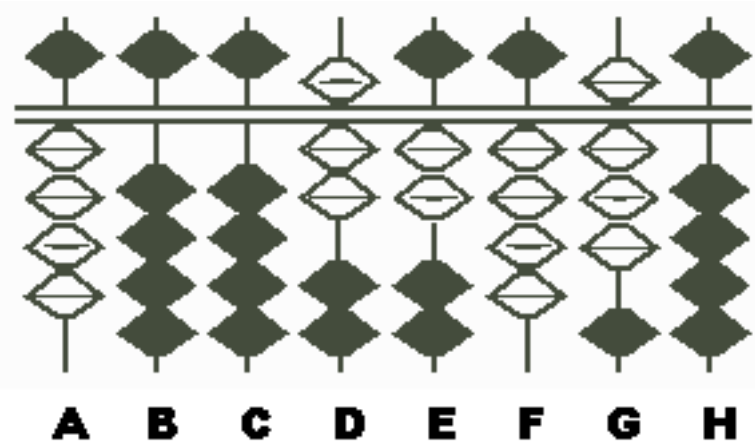
NOTE: As the 5 on F is in the tens place of the multiplicand, the product 20 must be set on GH.
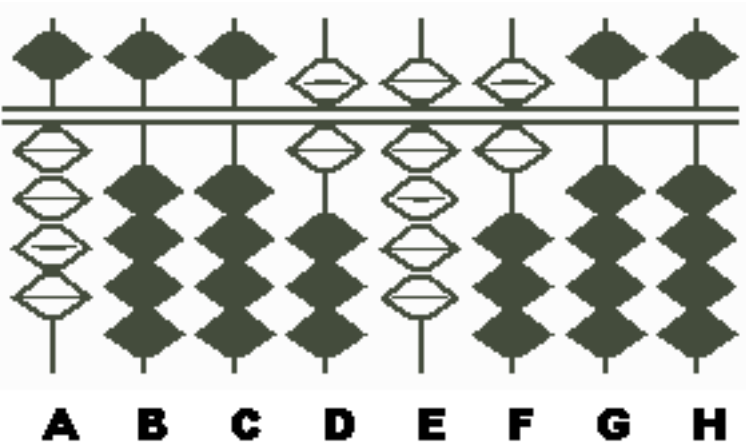
Step 3: Multiplying the 4 on A by the 2 on E, subtract the product 8 from the 8 on G. This leaves 724 on DEF. (Fig.3)

Fig.3

Step 4: Multiplying the 4 on A by the 7 on D, subtract the product 28 from 724 o DEF. This leaves 6,960 on DEFG (Fig.4). The answer is $6,960

Fig.4

- As written by Takashi Kojima
from his book, "Advances Abacus, Theory and Practice"

P.S. One more example can be found by following the link below;

Comp2.htm

Multiplication techniques on the Chinese Suan Pan, the "Extra Bead " and the "Suspended Bead" (contributed by Francis Wong and Totton Heffelfinger)

The Suan Pan, or Chinese abacus, was in use in China for three or four centuries before making its journey to Japan sometime in the 1600's. As Dave mentions in his introduction to the Handbook, the Soroban has gone through several incarnations. Unlike the present day Japanese Soroban which has evolved to become an instrument with a 1/4 bead construction, the Suan Pan has retained its classic 2/5 bead construction - 2 beads above the reckoning bar, 5 beads below. The techniques described below may offer some other ideas as to why this might be so.

These techniques are older and differ from modern day methods in that the order of multiplication is different. They might even be recognizable to an older generation in Japan, many of whom grew up learning older methods. New ideas and changes to the physical makeup of the Japanese abacus early in the last century brought about changes in the way many modern day Japanese do their abacus work. In learning these Chinese techniques, not only does it enable a better understanding of abacus use, it also provides a more in-depth look at the instrument and the thought behind its development and use.

1) The "Extra Bead" technique uses the top most heaven bead and has a value of 5. *(see Example 1: Step 3 below)*
2) The "Suspended Bead" technique also uses the top most heaven bead but in this case it's placed ½ way down the rod so that the bead neither touches the frame above nor the bead below. It has a value of 10. *(see Example 2: Steps 3 & 4 below)*

Before beginning these techniques, it would be useful to have a good understanding of both the addition and multiplication techniques described earlier in the main section of the Soroban/Abacus Handbook.

** Please note: When setting up the problems in the following examples, it's advisable to count the number of digits in the multiplier and leave the corresponding number of rods vacant on the right hand side of the Suan Pan. In doing so the unit number in the product will fall neatly on the last rod. In the examples below there are three numbers in the multiplier, so the last three rods on the right are left vacant.**

Example 1: "The Extra Bead"

When it comes to adding the products of the multiplication onto the abacus we use complementary numbers. For instance { 7 + 5 = 7 + (10 - 5) }. In the example below we'll use the same approach as much as possible. In doing so it helps to better focus on where the exclusive use of complementary numbers makes the job much more difficult.

189 x 576 = 108,864

**Step 1:** Set the multiplier 576 on DEF and the multiplicand 189 on HIJ. (Fig.1)

Fig. 1
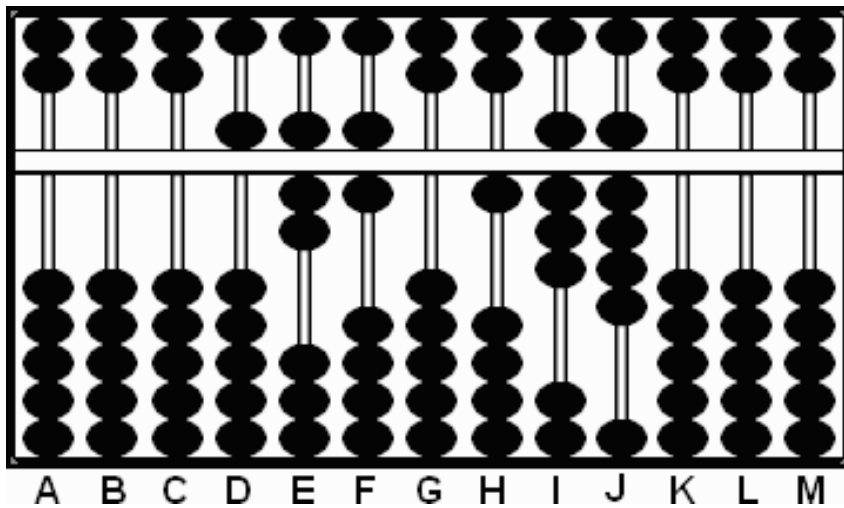
Step 1

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	5	7	6	0	1	8	9	0	0	0

Step 2:
a) Multiply the 9 on rod J by the 7 on E and add the product 63 on KL.
b) Multiply the 9 on rod J by the 6 on F and add the product 54 on LM
c) Multiply the same 9 on rod J by the 5 on rod D, remove the 9 on J and place the product 45 on JK . *This leaves 185184 on rods HIJKLM.* (Fig.2)

Fig. 2
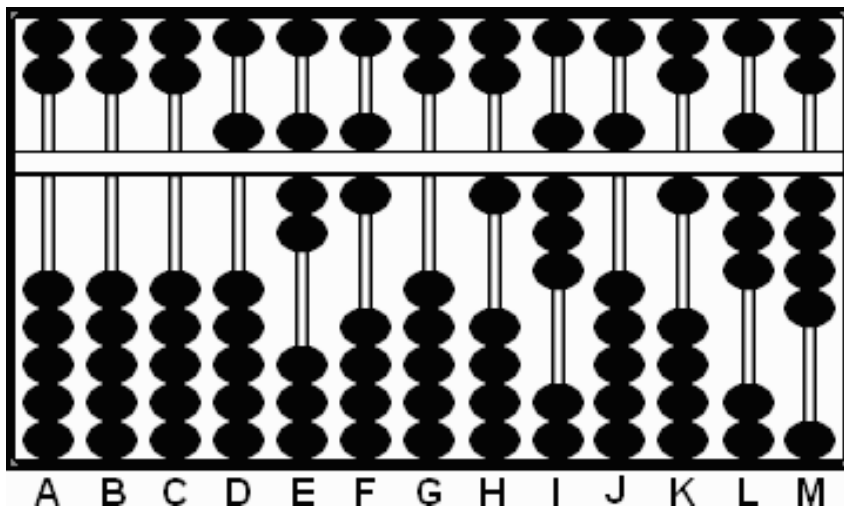
Step 2

A	B	C	D	E	F	G	H	I	J	K	L	M	
0	0	0	5	7	6	0	1	8	9	0	0	0	
										+ 6	3		step 2a
										+ 5	4		Step 2b
									(4)	5			Step 2c
0	0	0	5	7	6	0	1	8	5	1	8	4	

Step 3: This is where we use we use the "Extra Bead." At this point we're going to multiply the 8 on rod I by the 7 on E. If we were to do a carry here then we'd have have to change the 8 to 9. That would be confusing. (Especially since it's the 8 that's going to be multiplied by each of the digits in the multiplier.) Since there is an extra 5 bead available on J we'll use it to avoid confusion.

a) Multiply the 8 on rod I by the 7 on E and add the product 56 on JK, leaving 1 8 "10" 7 8 4 on rods HIJKLM. (Fig.3a)

Fig. 3a
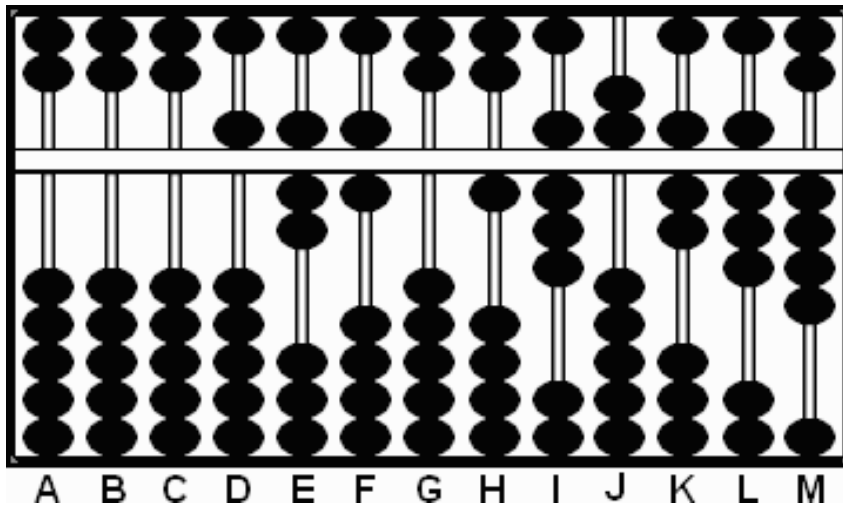

```
Step 3a
A B C D E F G H I J K L M
_____
0 0 0 5 7 6 0 1 8 5 1 8 4
                + 5 6       step 3a
_____
0 0 0 5 7 6 0 1 8"10"7 8 4
```

* It may be interesting to note that many Japanese Soroban made between circa 1850 and 1930 have one heaven bead and five earth beads. (i.e. 1 bead above the reckoning bar and 5 beads below.) I've always been confused as to why this might be. Learning from some of these older techniques may provide a clue. In the above step 3a one could, as an alternative, show the number 10 on rod J as one heaven bead down and 5 earth beads up.*

b) From here we can continue. Multiply the 8 on rod I by the 6 on F and add the product 48 on KL
c) Multiply the 8 on rod I by the 5 on D, remove the 8 on I and place the product 40 on rods IJ. This leaves 1 4 "11" 2 6 4 on rods HIJKLM. Next clear the 10 from rod J and carry it over to rod I changing the 4 to 5. This leaves 151264 on rods HIJKLM. (Fig.3b-3c)

Fig. 3b-3c
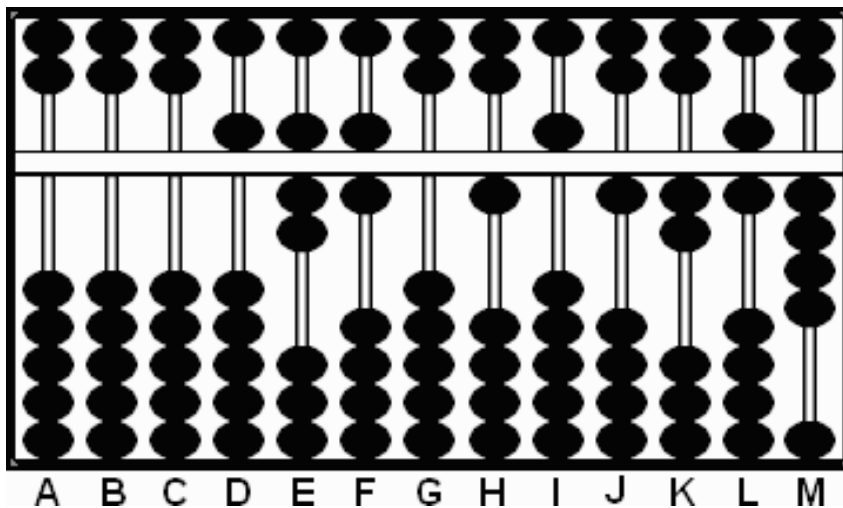

```
Steps 3b-3c
A B C D E F G H I J K L M
_____
0 0 0 5 7 6 0 1 8"10"7 8 4
                  + 4 8     step 3b
              + (4)0         step 3c
_____
0 0 0 5 7 6 0 1 5 1 2 6 4
```
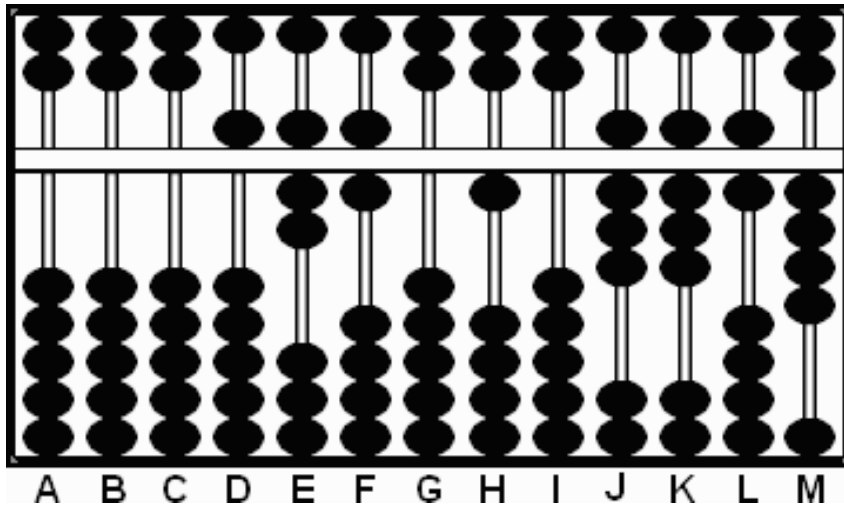
Step 4: Now we can continue using complements in the normal way.
a) Multiply the 1 on rod H by the 7 on rod E and add the product 7 on rod J
b) Multiply the 1 on rod H by the 6 on rod F and add the product 6 on rod K
c) Multiply the same 1 on rod H by the 5 on rod D, remove the 1 on rod H and add the product 5. This leaves 108,864 on rods HIJKLM, which is the answer. (Fig.4)

Fig. 4


```
Step 4
A B C D E F G H I J K L M
_____
0 0 0 5 7 6 0 1 5 1 2 6 4
                + 0 7         step 4a
                  + 0 6       step 4b
              + 0 5           step 4c
_____
0 0 0 5 7 6 0 1 0 8 8 6 4
```

Example 2: "The Suspended Bead" (Xuan - zhu)

In this next example we'll use a technique where we bring down the upper most 5 bead ½ way down the rod so that it neither touches the frame above nor the bead below. This "Suspended Bead" has a value of 10. Once again in solving this next example, we'll use complementary numbers as much as possible. However, in two instances it will be clear that having extra beads simplifies the process.

898 x 989 = 888, 122

Step 1: Set the multiplier 898 on DEF and the multiplicand 989 on HIJ. (Fig.1)
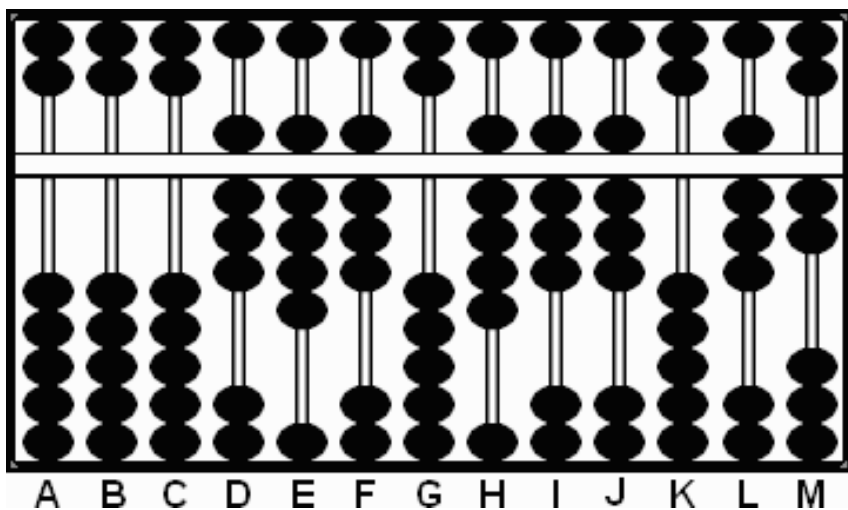
Fig. 1


```
Step 1
A B C D E F G H I J K L M
_____
0 0 0 8 9 8 0 9 8 9 0 0 0
```
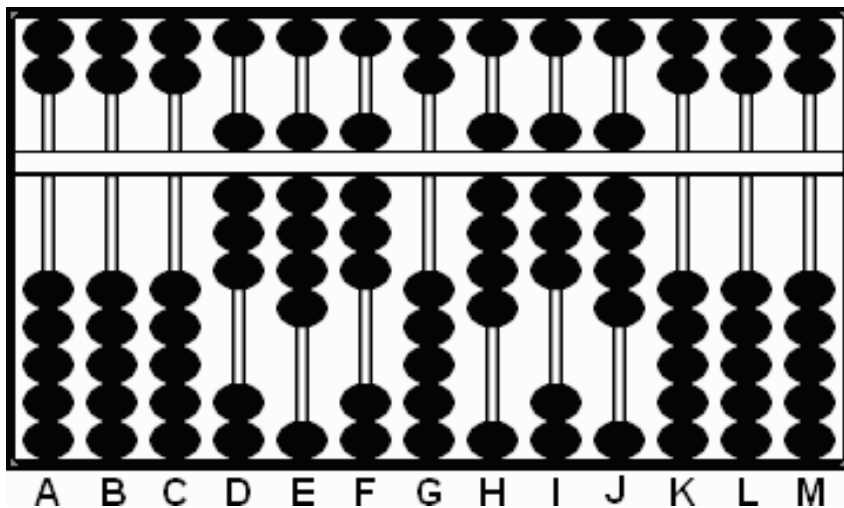
Step 2:
a) Multiply the 9 on rod J by the 9 on E and add the product 81 on KL.

b) Multiply the 9 on rod J by the 8 on F and add the product 72 on LM

c) Multiply the same 9 on rod J by the 8 on rod D, remove the 9 on J and place product 72 on JK . *This leaves 988082 on rods HIJKLM.* (Fig.2)

Fig. 2


```
Step 2
A B C D E F G H I J K L M
_____
0 0 0 8 9 8 0 9 8 9 0 0 0
                    + 8 1     step 2a
                      + 7 2   Step 2b
                    (7)2      Step 2c
_____
0 0 0 8 9 8 0 9 8 8 0 8 2
```

Step 3: This is where we use we use the "Suspended Bead." At this point we're going to multiply the 8 on rod I by the 9 on E. If we were to do a carry here then we'd have have to change the 8 to 9. That would be confusing. (Especially since it's the 8 that's going to be multiplied by each of the digits in the multiplier.) We might loose track. Instead we use the "Suspended Bead" technique.

a) Multiply the 8 on rod I by the 9 on E and add the product 72 on JK. By bringing down the suspended bead to signify 10 and subtracting 3 we've added 7 to rod J giving the rod a total value of 15.

b) Multiply the 8 on rod I by the 8 on F and add the product 64 on KL. This leaves a value of 9 8 "15" 9 2 2 on rods HIJKLM. (Fig. 3a-3b)
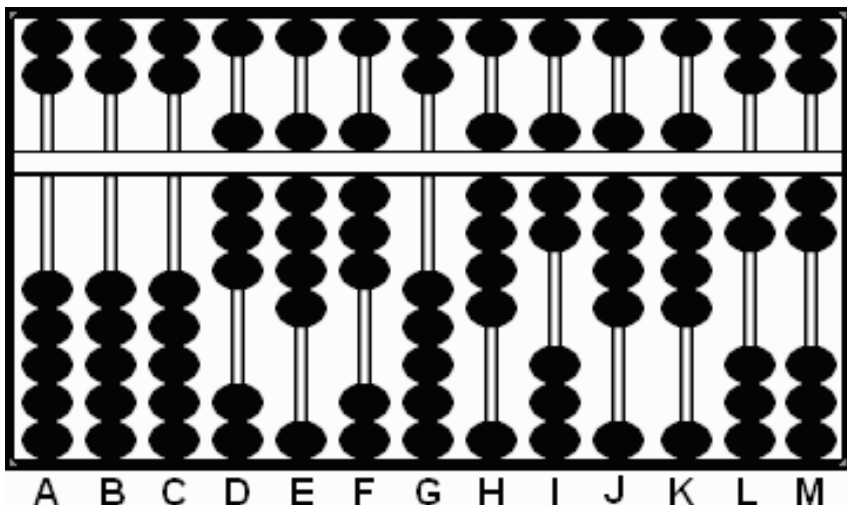
Fig. 3a-3b
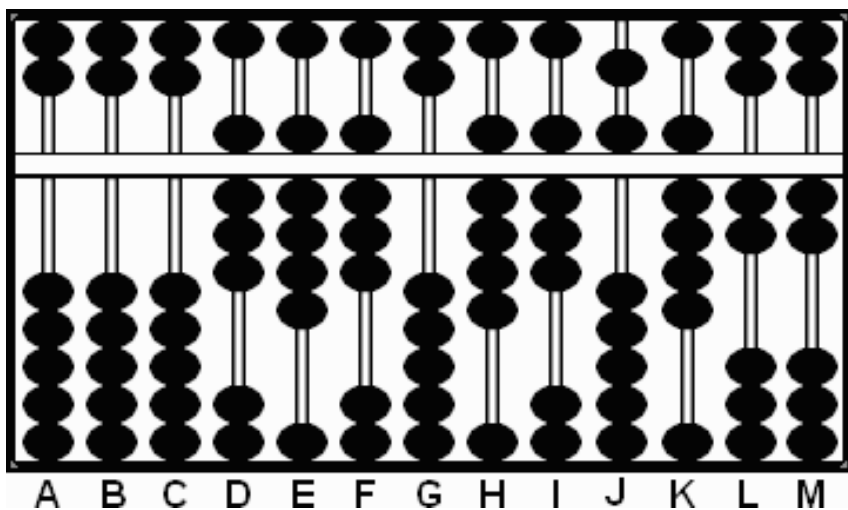

```
Step 3a-3b
A B C D E F G H I J K L M
_____
0 0 0 8 9 8 0 9 8 8 0 8 2
                      + 7 2      Step 3a
                        + 6 4    Step 3b
_____
0 0 0 8 9 8 0 9 8"15"9 2 2
```

c) Multiply the same 8 on rod I by the 8 on rod D, remove the 8 on I and place the product 64 on IJ . *This leaves 9 6*

"19" 9 2 2 on rods on rods HIJKLM. Next clear the "Suspended" 10 from rod J and carry it over to rod I changing the 6 to 7. This leaves 979922 on rods HIJKLM(Fig.3c)

Fig. 3c


```
Step 3c
A B C D E F G H I J K L M
_____
0 0 0 8 9 8 0 9 8"15"9 2 2
                   (6)4
_____
0 0 0 8 9 8 0 9 6 9 9 2 2    Step 3c
    + suspended 1 0
_____
0 0 0 8 9 8 0 9 7 9 9 2 2
```
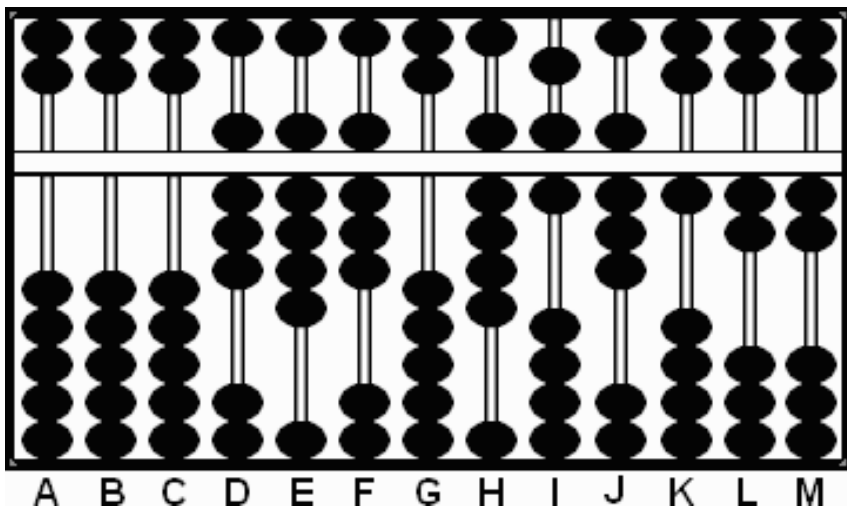
Step 4:
a) Multiply the 9 on H by the 9 on E and add the product 81 on IJ using the "Suspended" bead technique.
b) Multiply the 9 on H by the 8 on F and add the product 72 on JK. This leaves 9 "16" 8 1 2 2 on HIJKLM. (Fig.4a-4b)
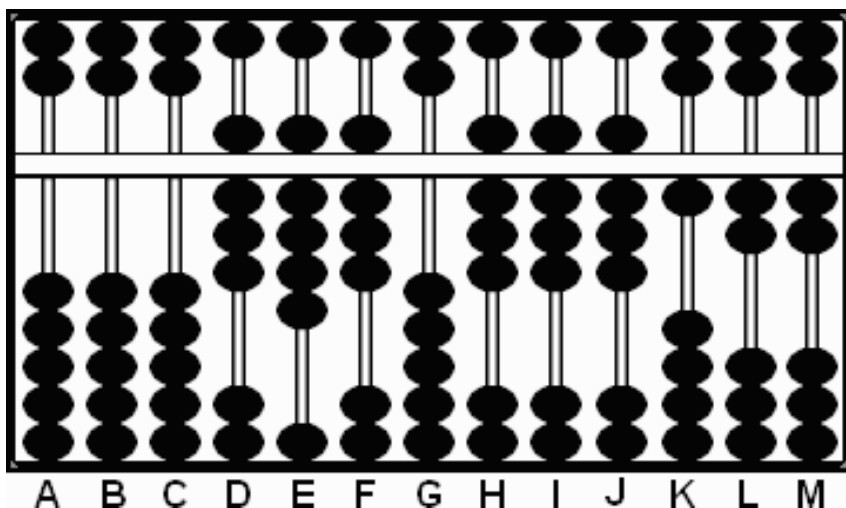
Fig. 4a-4b


```
Step 4a-4b
A B C D E F G H I J K L M
_____
0 0 0 8 9 8 0 9 7 9 9 2 2
                + 8 1          Step 4a
                  + 7 2        Step 4b
_____
0 0 0 8 9 8 0 9"16"8 1 2 2
```

c) Multiply the 9 on H by the 8 on D, remove the 9 on H and place the product 72 on HI. This leaves 7 "18" 8 1 2 2 on rods HIJKLM. Next clear the "Suspended" 10 from rod I and carry it over to rod H changing the 7 to 8. This leaves 888122 on rods HIJKLM, which is the answer. (Fig.4c)

Fig. 4c


```
Step 4c
A B C D E F G H I J K L M
_____

0 0 0 8 9 8 0 9"16"8 1 2 2
                (7)2
_____
0 0 0 8 9 8 0 7 8 8 1 2 2    Step 4c
  + suspended 1 0
_____
0 0 0 8 9 8 0 8 8 8 1 2 2
```

Over the centuries, both the Japanese and the Chinese have developed and used many different techniques for solving problems of multiplication on the abacus. One widely used method is the one taught by Takashi Kojima in his book, "The Japanese Abacus, its use and Theory" published in 1954 . But when introducing this technique, even he says, "There are several methods of multiplication on the abacus. The one introduced in the following pages is a recent method...".

Even though it's possible to solve multiplication problems while using these Chinese techniques on a 1/4 bead Soroban, the above examples show the answers come more easily with the help of extra beads.

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![abacus] Short Division techniques on the Chinese Suan Pan

Below is a list of rules. They may look a little complicated at first glance but they are easily understood. As such there should be no need to commit them to memory. (See below for a brief explanation.)

LIST OF TRADITIONAL RULES

1/1 = forward 1

1/2 = 5
2/2 = forward 1

1/3 = 3 plus 1
2/3 = 6 plus 2
3/3 = forward 1

1/4 = 2 plus 2
2/4 = 5
3/4 = 7 plus 2
4/4 = forward 1

1/5 = 2
2/5 = 4
3/5 = 6
4/5 = 8
5/5 = forward 1

1/6 = 1 plus 4
2/6 = 3 plus 2
3/6 = 5
4/6 = 6 plus 4
5/6 = 8 plus 2
6/6 = forward 1

1/7 = 1 plus 3
2/7 = 2 plus 6
3/7 = 4 plus 2
4/7 = 5 plus 5
5/7 = 7 plus 1
6/7 = 8 plus 4
7/7 = forward 1

1/8 = 1 plus 2
2/8 = 2 plus 4
3/8 = 3 plus 6
4/8 = 5
5/8 = 6 plus 2
6/8 = 7 plus 4
7/8 = 8 plus 6
8/8 = forward 1

1/9 = 1 plus 1
2/9 = 2 plus 2
3/9 = 3 plus 3
4/9 = 4 plus 4
5/9 = 5 plus 5
6/9 = 6 plus 6
7/9 = 7 plus 7
8/9 = 8 plus 8
9/9 = forward 1

** Please note: In the above set of rules.**
"plus 1" means adding 1 to the column immediately to the right, "plus 2" means adding 2 and so on...
"forward 1" means adding 1 to the column immediately to the left.

Explanation of Rules:

Using a few random examples, this is how I think of it;

a) *2 /3 = 6 plus 2*   ==>   20 ÷ 3 = 6 (place remainder 2) on the column to the right.

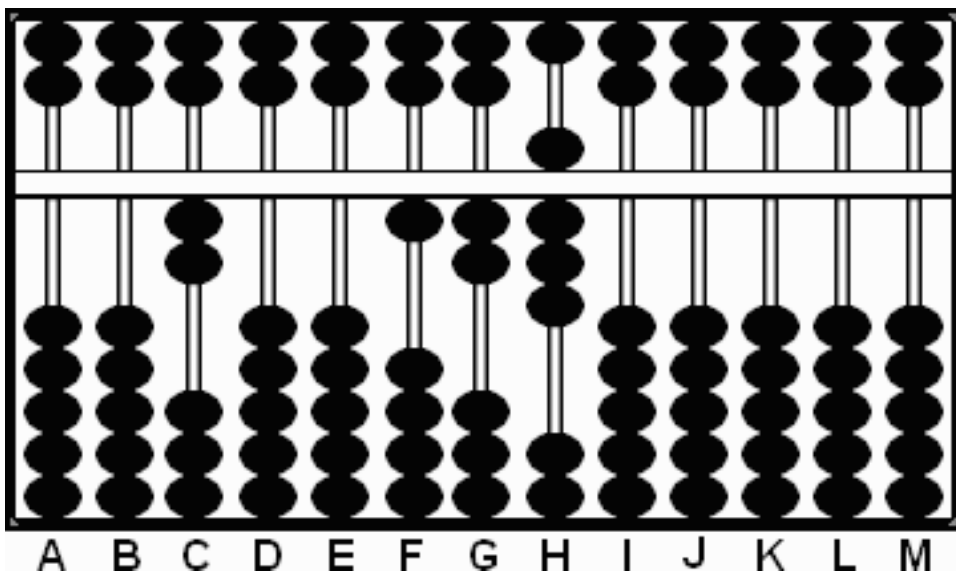b) *6 /7 = 8 plus 4*   ==>   60 ÷ 7 = 8 (place remainder 4) on the column to the right.

c) *5 /5 = forward 1* ==>   5 ÷ 5 = 1 (forward 1) on the column to the left.

All is best explained using examples.

Example 1: 128 ÷ 2 = 64

Step 1: Set the dividend 128 on FGH and the divisor 2 on C. (Fig.1)
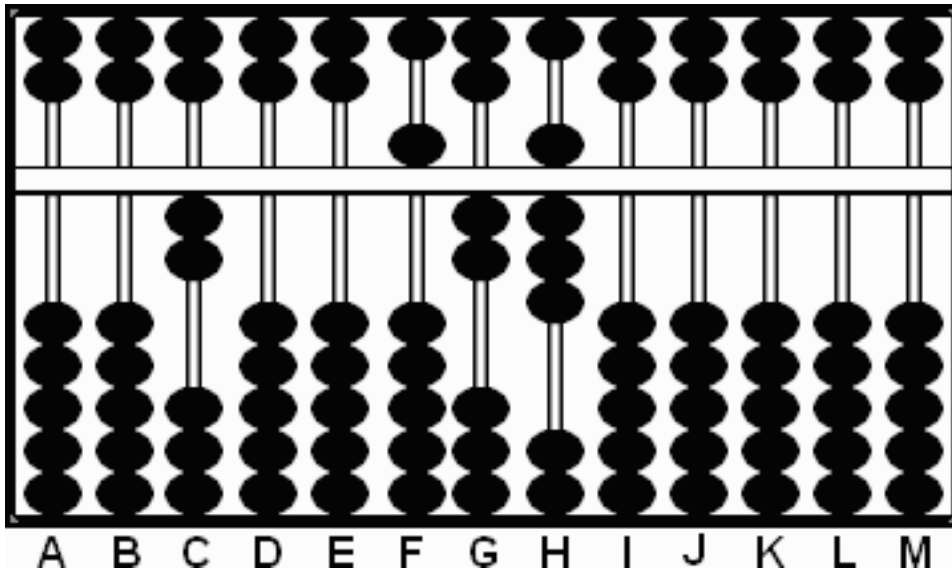

Fig. 1


```
Step 1
A B C D E F G H I J K L M
_____
0 0 2 0 0 1 2 8 0 0 0 0 0
```

Step 2: Following *rule 1 /2 = 5*   ===> divide 2 on rod C into 1 on F. (Think of it in terms of 10 ÷ 2 = 5). Remove the 1 and replace it with the quotient 5 on rod F, leaving 528 on rods FGH. (Fig.2)
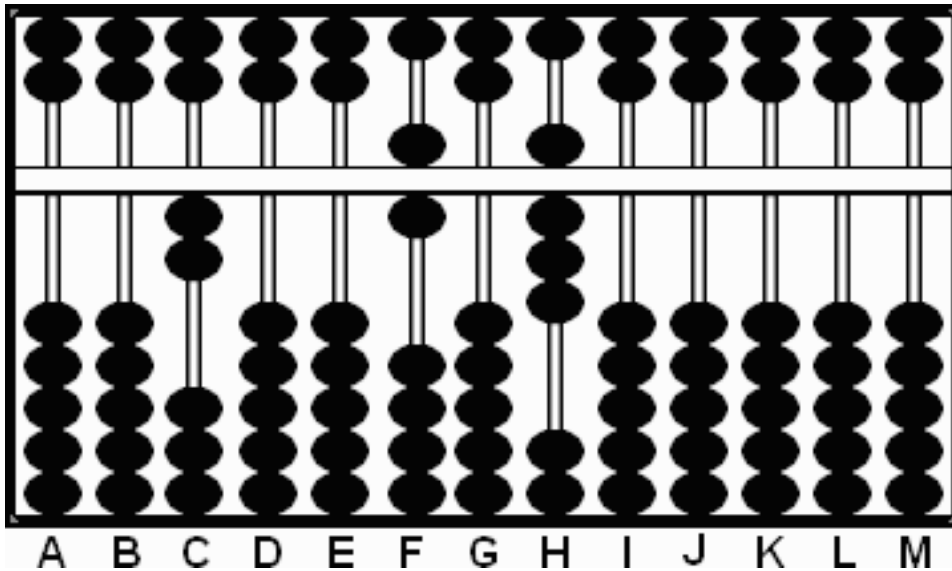
Fig. 2

Step 2

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	7	0	0	1	2	8	0	0	0	0	0
					(5)							
0	0	7	0	0	5	2	8	0	0	0	0	0

Step 3: Following *rule 2 /2 = forward 1* ===> this means clear the 2 on rod G and add 1 to the rod on the left. Therefore, 5 on rod F becomes 6, leaving 608 on rods FGH. (Fig.3)

Fig. 3

Step 3
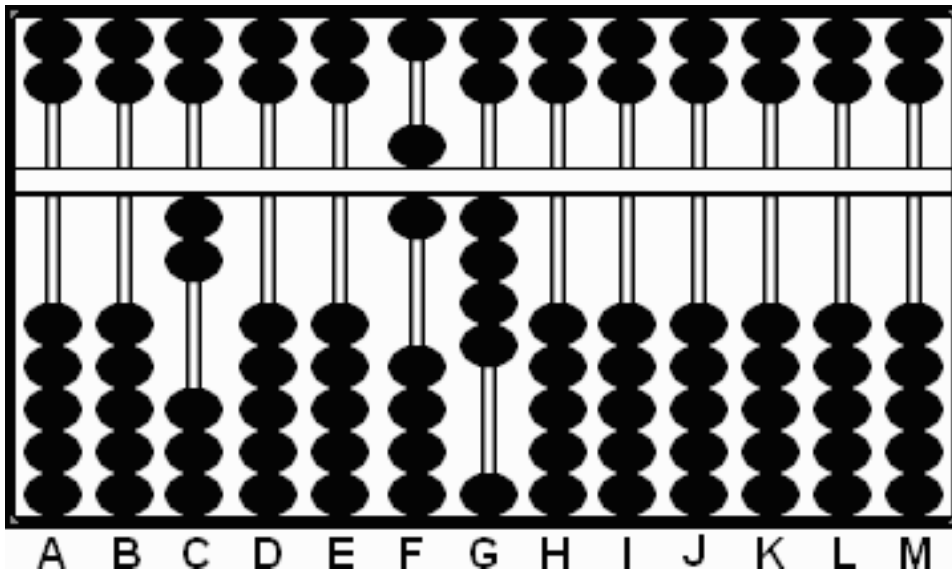
A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	7	0	0	5	2	8	0	0	0	0	0
clear						(2)						
forward					1							
0	0	7	0	0	6	0	8	0	0	0	0	0

Step 4: Following ** *rule 8 / 2 = forward 4* ** ===> this means clear the 8 on rod H and add 4 to the rod on the left. Therefore, 0 on rod G becomes 4, leaving 64 on rods FG, which is the answer. (Fig.4) ** This is just *rule 2 /2 = forward 1* that's been applied four times **
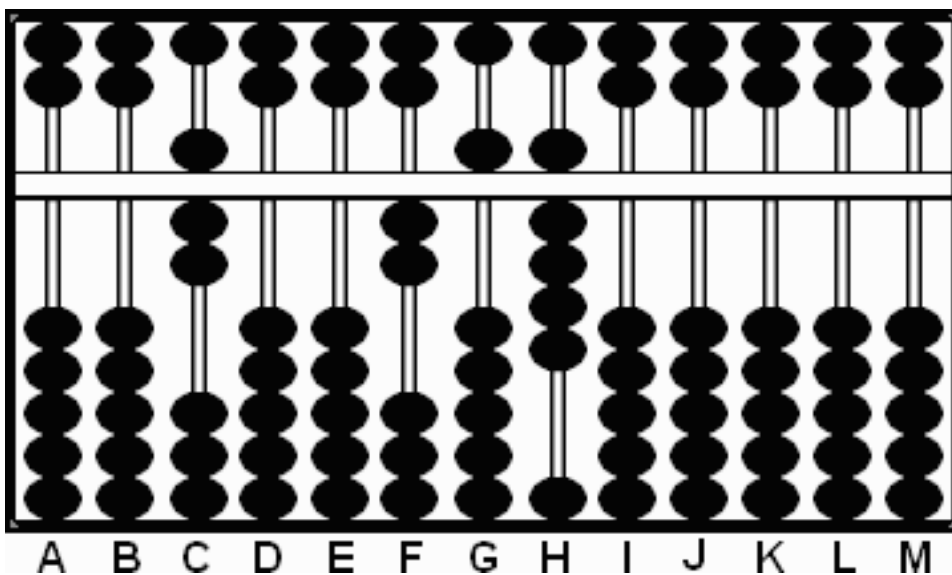
Fig. 4

Step 3

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	7	0	0	6	0	8	0	0	0	0	0

clear   (8)

forward   4

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	7	0	0	6	4	0	0	0	0	0	0

Example 2: 259 ÷ 7 = 37

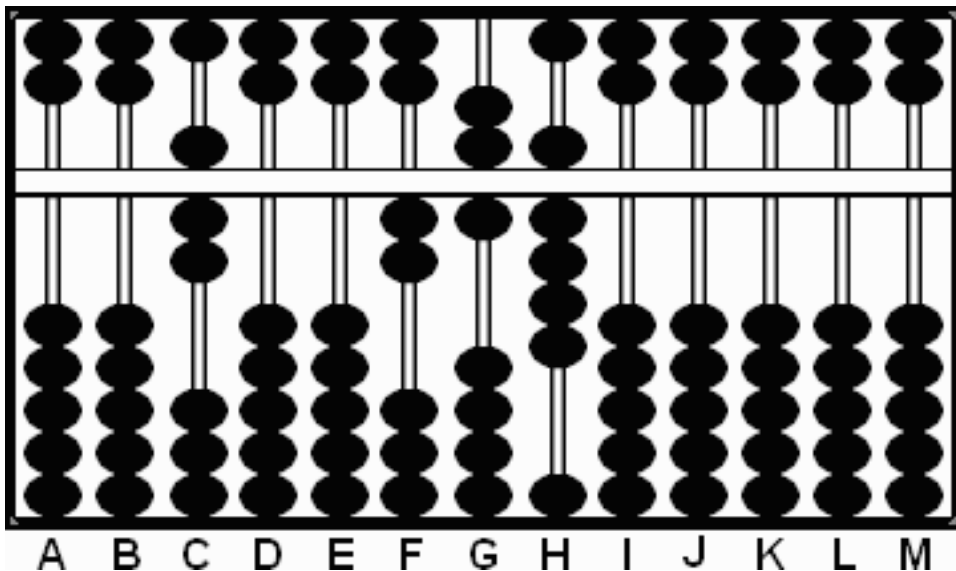**Step 1:** Set the dividend 259 on FGH and the divisor 7 on C. (Fig.1)

Fig. 1

Step 1

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	7	0	0	2	5	9	0	0	0	0	0

Step 2: Step 2: Following *rule 2 /7 = 2 plus 6*   ===> divide 7 on rod C into 2 on F. (Think of it in

terms of 20 ÷ 7 = 2 with a remainder of 6). Place the quotient 2 on rod F and place the remainder 6 on G, leaving 2 "11" 9 on rods FGH. (Fig.2)
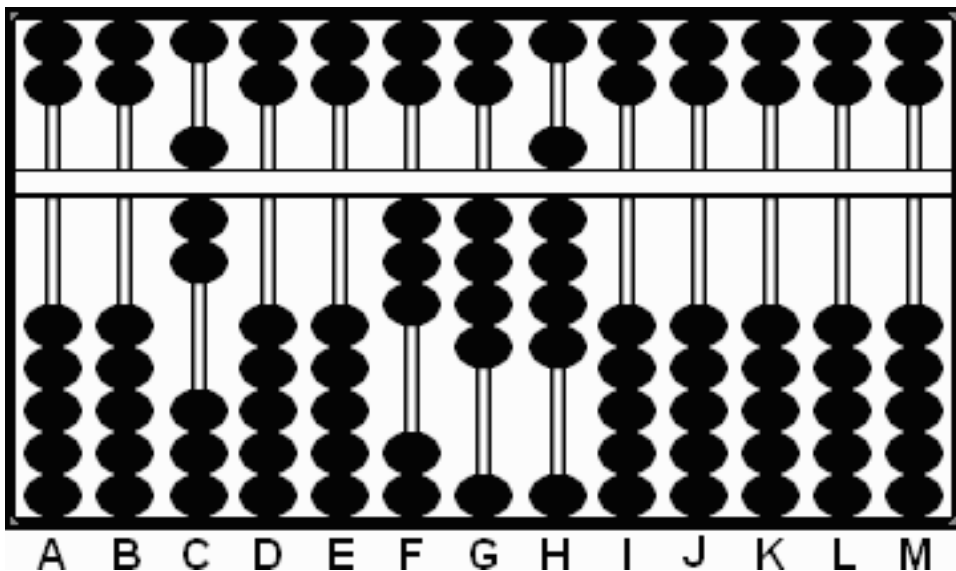
Fig. 2


```
Step 2
A B C D E F G H I J K L M
─────────────────────────────
0 0 7 0 0 2 5 9 0 0 0 0 0
            (2)
plus        + 6
─────────────────────────────
0 0 7 0 0 2"11"9 0 0 0 0 0
```

Step 3: The 11 on rod G is a number greater than 7. So take 7 away from rod G and *forward 1* onto rod F changing the 2 to a 3. This leaves 349 on rods FGH. (Fig.3)
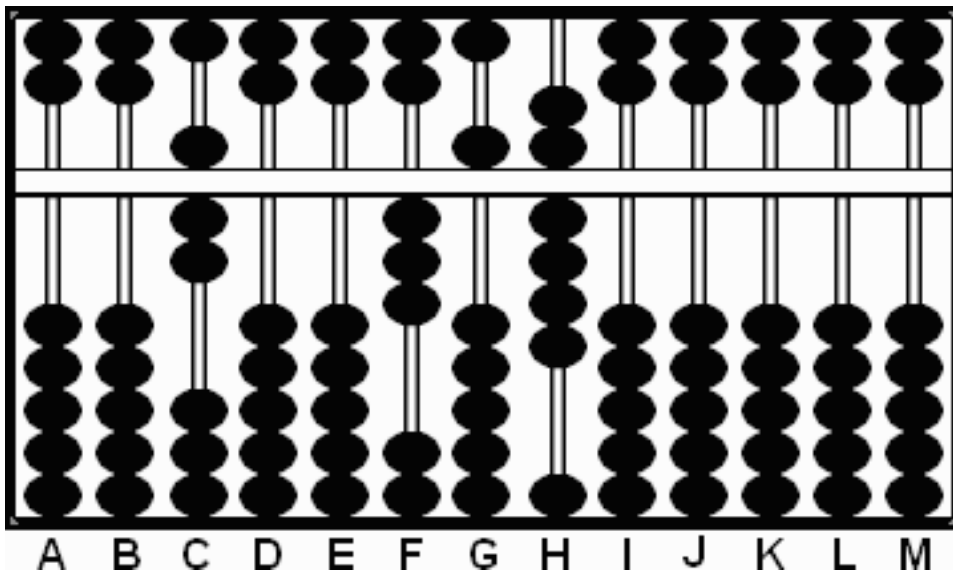
Fig. 3


```
Step 3
A B C D E F G H I J K L M
─────────────────────────────
0 0 7 0 0 2"11"9 0 0 0 0 0
              - 7
forward       1
─────────────────────────────
0 0 7 0 0 3 4 9 0 0 0 0 0
```

Step 4: Following *rule 4 /7 = 5 plus 5* ===> (Think of it in terms of 40 ÷ 7 = 5 with a remainder of 5). Replace the 4 on G with the quotient 5. Place the remainder 5 onto rod H, leaving 3 5 "14" on rods FGH. (Fig.4)
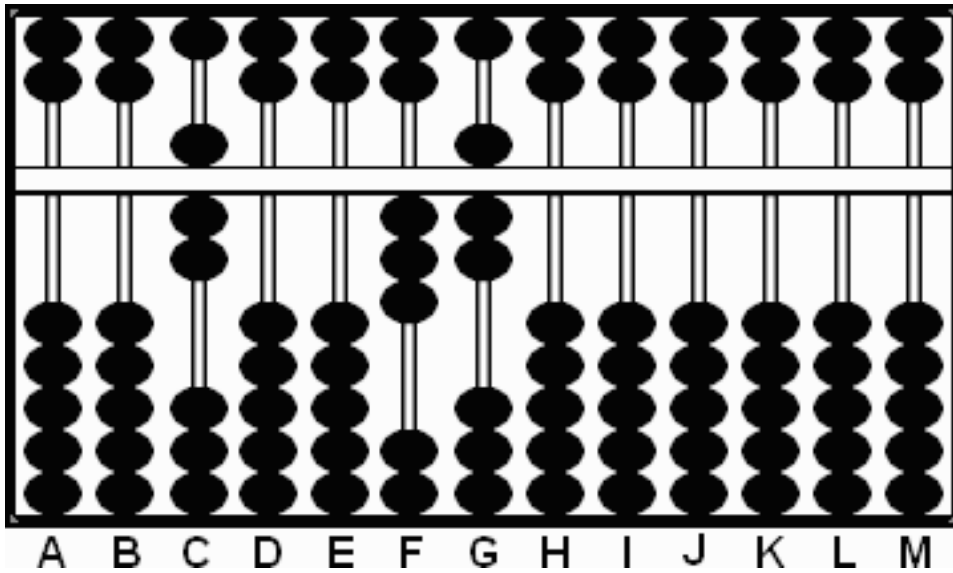
Fig. 4

```
Step 4
A B C D E F G H I J K L M
_____

0 0 7 0 0 3 4 9 0 0 0 0 0
                (5)
plus              5
_____
0 0 7 0 0 3 5"14"0 0 0 0 0
```

Step 5: Following   ** *rule 14 /7 = forward 2* **   ===> this means clear the 14 on rod H and add 2 to the rod on the left. Therefore, 5 on rod G becomes 7, leaving 37 on rods FG, which is the answer. (Fig.5) ** This is just   *rule 7 /7 = forward 1*   that's been applied two times **

Fig. 5


```
Step 5
A B C D E F G H I J K L M
_____

0 0 7 0 0 3 5"14"0 0 0 0 0
clear        ("14")
forward       2
_____
0 0 7 0 0 3 7 0 0 0 0 0 0
```

It's evident that some of the above steps could have been done more efficiently. For instance in Example 2: Step 3, instead of dividing 7 into the 4 on rod G, we could have looked ahead and noticed 49 on GH. At this point dividing 7 into 49 would have allowed us to come to our answer of 37 that much more quickly. I took the longer apnroach only because it may be of help to see some of the thinking behind the process.

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▦ Long Division techniques on the Chinese Suan Pan

The following is an example for learning long division using a Chinese method. In order to solve the problem one needs to understand two techniques;

1) The "Extra Bead Technique" which uses the top most heaven bead and has a value of 5. *(see Step 2a below)*
2) The "Suspended Bead Technique" which also used the top most heaven bead but in this case it is set ½ way down the rod so that the bead neither touches the frame above nor the bead below. It has a value of 10. *(see Step 3a below)*

These Chinese bead techniques were taught to me by Francis Wong. The actual long division technique is one taught to me by Torsten Reincke.
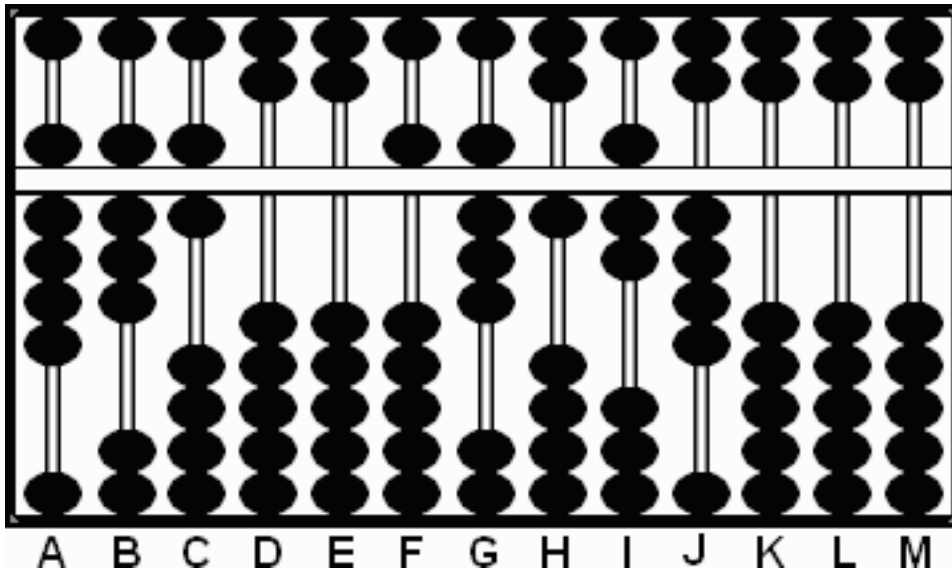
The example below uses the following rules;
a) *rule 5 /9= 5 plus5*
b) *rule 8 /9= 8 plus 8*
c) *rule 9 /9= forward 1*

For a more complete list of rules and how to use them please link to an earlier chapter called: [Chinese Short Division techniques](#)

Example: 58174 ÷ 986 = 59

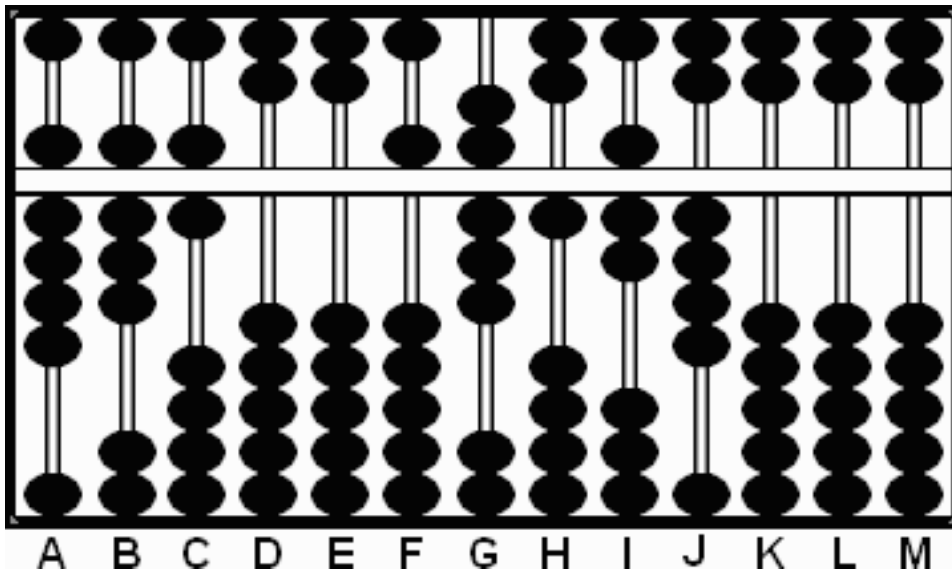**Step 1:** Set the dividend 58174 on rods FGHIJ and the divisor 986 on ABC. (Fig.1)

Fig. 1

Step 2:

a) Following *rule 5 /9= 5 plus 5* ===> place the quotient 5 on rod F and the remainder 5 on G, leaving 5 "13" 1 7 4 on FGHIJ. *(Note: This is where we use the *The Extra Bead Technique* which allows for 13 to be placed on rod G.)* (Fig.2)
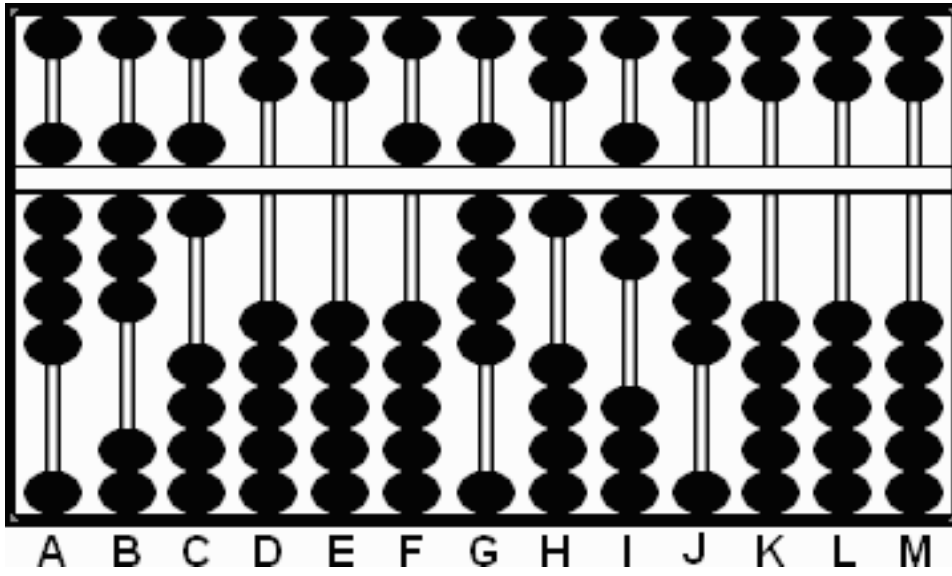
Fig. 2a

Step 2a

A B C D E F G H I J K L M
```
9 8 6 0 0 5 8 1 7 4 0 0 0
plus          5
```
```
9 8 6 0 0 5"13"1 7 4 0 0 0
```

b) Multiply 5 on F by 8 on B. Subtract the product 40 from rods GH, leaving, 59174 on rods FGHIJ. (Fig.2b)
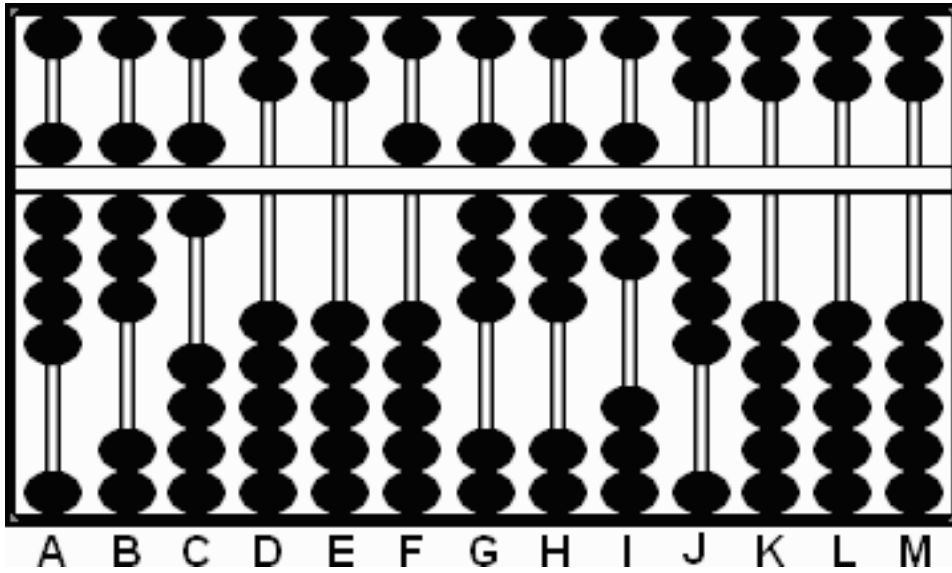
Fig. 2b

Step 2b

A	B	C	D	E	F	G	H	I	J	K	L	M
9	8	6	0	0	5	"13"	1	7	4	0	0	0
						− 4	0					
9	8	6	0	0	5	9	1	7	4	0	0	0

c) Multiply 5 on F by 6 on C. Subtract the product 30 from rods GH, leaving, 58874 on rods FGHIJ. (Fig.2c)

Fig. 2c

Step 2c

A	B	C	D	E	F	G	H	I	J	K	L	M
9	8	6	0	0	5	9	1	7	4	0	0	0
						− 3	0					
9	8	6	0	0	5	8	8	7	4	0	0	0

Step 3:

a) Following *rule 8 /9= 8 plus 8*   ===> place the quotient 8 on rod G and the remainder 8 on H. *(Note: Since rod H already carries a large number we don't have 8 available. This is where we'll use*

The Suspended Bead Technique *which counts for 10.)* Slide the top most bead **½** way down on rod H for a value of 10 then subtract 2 giving us the required 8. This leaves 5 8 "16" 7 4 on rods FGHIJ.

(Fig.3a)

Fig. 3a
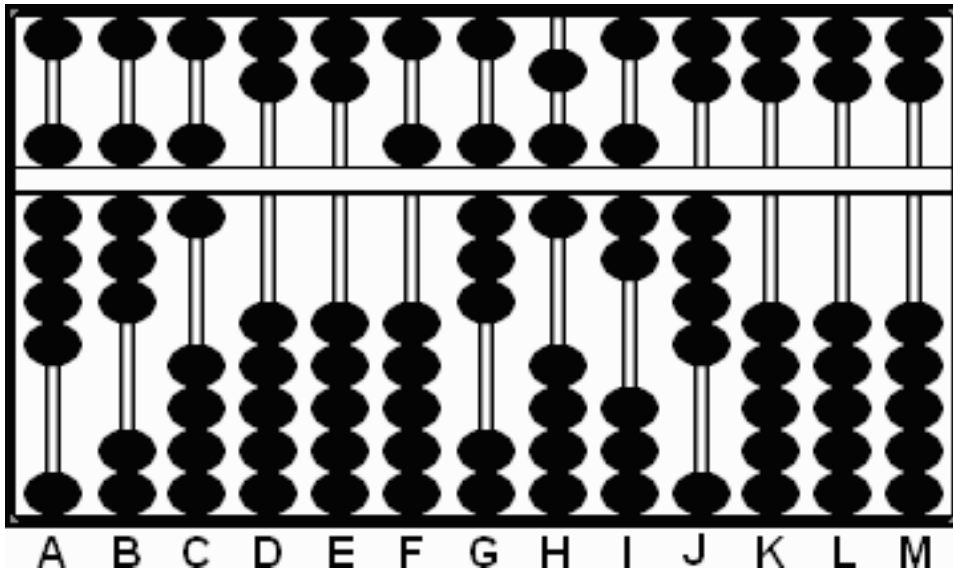
Step 3a

A	B	C	D	E	F	G	H	I	J	K	L	M
9	8	6	0	0	5	8	8	7	4	0	0	0
suspended							"10"					
							− 2					

A	B	C	D	E	F	G	H	I	J	K	L	M
9	8	6	0	0	5	8	"16"	7	4	0	0	0

b) Multiply the 8 on G by the 8 on B and subtract the product 64 from rods HI, leaving 5 8 "10" 3 4 on FGHIJ. (Fig.3b) ***see a variation below***

Fig. 3b

Step 3b

A	B	C	D	E	F	G	H	I	J	K	L	M
9	8	6	0	0	5	8	"16"	7	4	0	0	0
							−	6	4			

A	B	C	D	E	F	G	H	I	J	K	L	M
9	8	6	0	0	5	8	"10"	3	4	0	0	0

c) Multiply the 8 on G by the 6 on C and subtract the product 48 from rods IJ, leaving 58986 on rods FGHIJ. (Fig.3c)
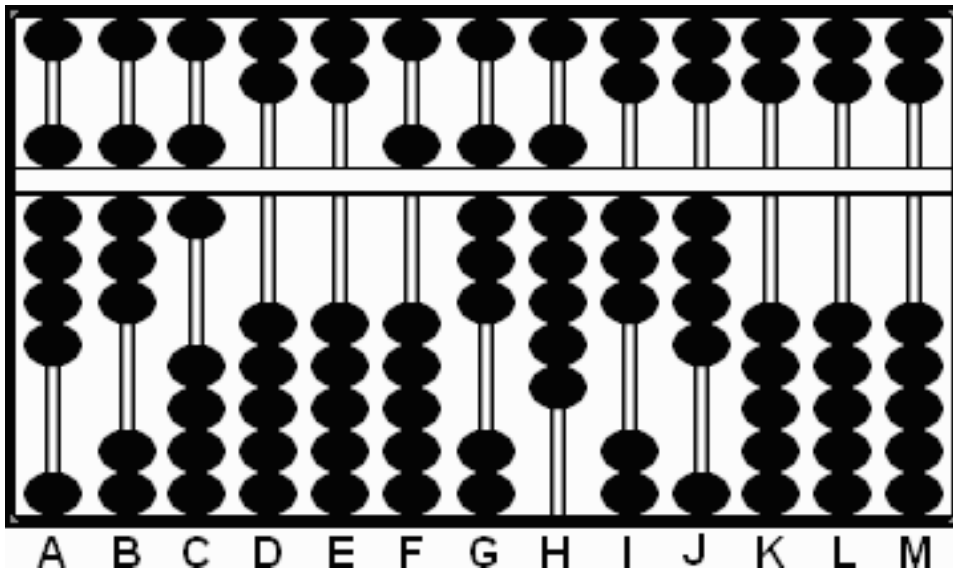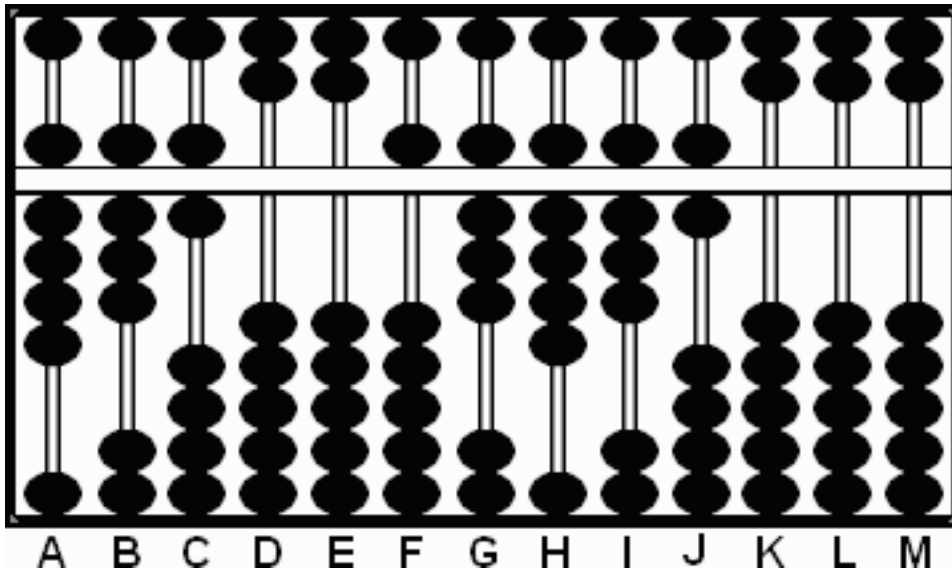
Fig. 3c

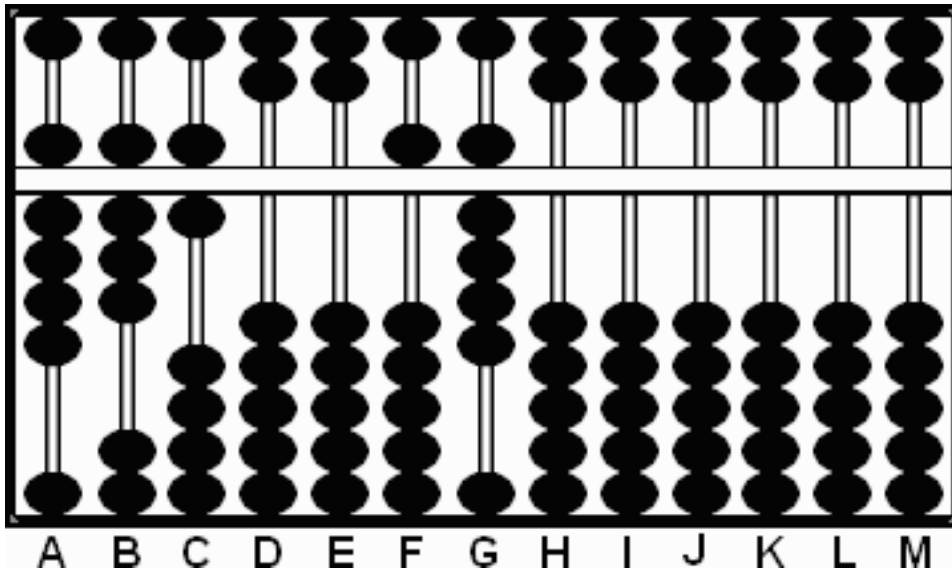**Step 3c**

A	B	C	D	E	F	G	H	I	J	K	L	M
9	8	6	0	0	5	8	"10"	3	4	0	0	0
						–		4	8			
9	8	6	0	0	5	8	9	8	6	0	0	0

d) Here we make a revision. Changing the 8 on G to a 9 we can now subtract a further 986 from rods HIJ. This leaves 59 on rods FG which is the answer. (Fig.3d)

Fig. 3d

Step 3d

A	B	C	D	E	F	G	H	I	J	K	L	M
9	8	6	0	0	5	9	0	5	4	0	0	0
							–	5	4			
9	8	6	0	0	5	9	0	0	0	0	0	0

*** In the above step 3b one could have exercised the option to do a revision using *rule 9 /9= forward 1*   ===> thereby changing the 8 on rod G to a 9. Multiplying the revised 9 on G by the 8 on B followed by the 6 on C and subtracting the products from rods HIJ, one could have come to the answer in a slightly

different way.***

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